Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Compute the eigenvalues and eigenfunctions for boundary value problem :

$$y'' + 2y' + (1 - \lambda)y = 0;\ y(0) = 0 \text{ and } y(1) = 0$$

Also prove that the set of eigenfunctions for the given problem is an orthogonal set.

11. Determine the curve of prescribed length $2l$ which joins the points $(-a, b)$ and (a, b) and has its center of gravity as low as possible.

12. Prove that :

(i) If $a + b + c > 0$, then :

$$\lim_{x\to 1}\{(1-x)^{a+b-c}\,{}_2F_1(a,\ b;\ c;\ x)\} = \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}$$

(ii) $${}_2F_1(a, b; 1 - a + b; -1) = \frac{\Gamma(1-a+b)\Gamma\left(1+\frac{b}{2}\right)}{\Gamma(1+b)\Gamma\left(1+\frac{b}{2}-a\right)}$$

13. Prove that :

(i) $$\int_0^\infty e^{-st} L_n(t)\,dt = \frac{1}{s}\left(1-\frac{1}{s}\right)^n$$

(ii) $$\frac{1}{(1-t)^{k+1}}\exp\left\{-\frac{xt}{1-t}\right\} = \sum_{n=0}^{\infty} L_n^k(x)t^n$$

MAMT–03/MSCMT–03

June – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variations and Special Functions)

Paper : MAMT–03/MSCMT–03

*Time : **3** Hours*] [*Maximum Marks :* ***80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) What is the dimension of the following differential equation ?

$$x\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+\frac{dy}{dx}=4$$

(ii) Write down the necessary and sufficient condition for the integrability of the total differential equation $P dx + Q dy + R dz = 0$.

(iii) Classify the following PDE as Hyperbolic, Parabolic or Elliptic :

$$\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$$

(iv) Define Functional.

(v) Write Euler Lagrange's equation for the stationary value of the integral :

$$I=\int_{x_1}^{x_2} f(x,y,y',y'',y''')dx$$

(vi) Check whether the boundary value problem :

$$y'' + \lambda y = 0;\ y'(-\pi) = 0;\ y'(\pi) = 0$$

is a Strum-Liouville problem for $\lambda < 0$.

(vii) Write Orthogonal property for Legendre polynomial.

(viii) Vandermonde's theorem ${}_2F_1(-n, b; c; 1) = \ldots$ … … . where n is a positive integer.

(Fill in the blank)

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Solve :

$$z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx)dz = 0$$

3. Find the general solution of the Riccati's equation :

$$\frac{dy}{dx}=2-2y+y^2$$

4. A tightly stretched string which has fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = k\sin^3\left(\frac{\pi x}{e}\right)$. It is released from rest from its position. Find the displacement $y(x, t)$.

5. Define Gauss's Hypergeometric series and discuss its convergence.

6. Prove that the eigenvalues of Sturm-Liouville system are real.

7. Show that :

$$(1-2xh+h^2)^{-1/2}=\sum_{n=0}^{\infty}P_n(x)h^n.\ |x|\le 1,\ |h|\le 1$$

8. Solve the Legendre's equation :

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

9. Expand x^n in a series of Hermite polynomials.