10. Let $< f_n >$ be a sequence of bounded measurable functions defined on a set E of finite measure. If there exists a positive number M such that $|f_n(x)| \leq M$ for all $n \in N$ and for all $x \in E$ and if $< f_n >$ converges in measure to a bounded measurable function f on E, then prove that :

$$\lim_{n \to \infty} \int_E f_n(x)dx = \int_E f(x)dx$$

11. (i) Show that intersection of two measurable sets is also a measurable set.

    (ii) Prove that two closed subsets of a topological space are separated iff they are disjoint.

12. Prove that every interval is measurable.

13. Prove that a series $\sum_{i=1}^{\infty} f_i$ of pairwise orthogonal elements in L_2 is converges iff the series of real numbers $\sum_{i=1}^{\infty} \| f_i \|^2$ is convergent.

MAMT–02/MSCMT–02

June – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Real Analysis and Topology)

Paper : MAMT-02/MSCMT-02

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define σ ring.

   (ii) Define measurable function.

   (iii) State Riesz-Fisher theorem.

   (iv) State Minkowski's inequality.

   (v) Write the necessary and sufficient conditions for a bounded function f defineed on the interval $[a, b]$, to be L-integrable.

   (vi) Define Hilbert space.

   (vii) Define exterior of a set.

   (viii) What do you mean by filter base ?

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Let $\{E_n\}$ be a countable collection of sets of real numbers, then show that :

$$m^*\left(\bigcup_n E_n\right) \le \sum_n m^*(E_n)$$

3. If f is a bounded measurable function defined on a measurable set E, then prove that $|f|$ is L-integrable over E and :

$$\left|\int_E f(x)dx\right| \le \int_E |f(x)|dx$$

4. Show that every bounded measurable function f defined on a measurable set E is L-integrable.

5. Show that the L^p-space is a linear space.

6. State and prove Holder's inequality.

7. Prove that in a T_2-space, a convergent sequence has a nunique limit.

8. Prove that subset of real numbers R is connected if and only if it is an interval.

9. Prove that the product space (X × Y, W) is compact if and only if each of the spaces (X, τ) and (X, V) is compact.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.