405

MA/MSCMT–10

June – Examination 2020

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Mathematical Programming)

Paper : MA/MSCMT-10

Time : 3 Hours] [*Maximum Marks : 80*

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator/simple calculator allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Supporting Hyperplane.

(ii) Define bounded variable linear programming problem.

(iii) Gomory's method to solve integer programming problem is called a cutting plane method, why ?

(iv) Distinguish between pure and mixed integer programming.

(v) Write the following quadratic form Q(x) in the matrix form where :

$$Q(x) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$$

(vi) Define saddle point.

(vii) Define convex programming problem.

(viii) Define a general non-linear programming problem.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that a positive semi-definite quadratic form $f(X) = X^T AX$ is a convex function over R^n.

3. Find the optimum integer solution to the following linear programming problem :

Maximize : $Z = x_1 + 2x_2$

s.t. :

$$2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

x_1, x_2 are integers and ≥ 0.

4. Use Lagrangian function to find the optimal solution of the following non-linear programming problem :

Maximize :

$$f(x) = -3x_1^2 - 4x_2^2 - 5x_3^2$$

Subject to :

$$x_1 + x_2 + x_3 = 10$$

$$x_1, x_2, x_3 \geq 0$$

5. Solve the following programming problem graphically and verify the Kuhn-Tucker condition for the same :

Maximize :

$$f(x_1, x_2) = 2x_1 + 3x_2$$

Subject to :

$$x_1^2 + x_2^2 \leq 20$$

$$x_1 x_2 = 8$$

$$x_1, x_2 \geq 0$$

6. Solve the following linear programming problem with the help of revised simplex method but without use of artificial variables :

Maximize :

$$Z = 2x_1 - 6x_2$$

s.t. :

$$x_1 - 3x_2 \leq 6$$

$$2x_1 + 4x_2 \geq 8$$

$$-x_1 + 3x_2 \leq 6,$$

$$x_1 x_2 \geq 0$$

7. Solve the following linear programming problem by dynamic programming :

Maximize :

$$Z = 8x_1 + 7x_2$$

Subject to :

$$2x_1 + x_2 \le 8$$

$$2x_1 + 2x_2 \le 15$$

and $$x_1, x_2 \ge 0$$

8. Use dynamic programming to solve the following problem :

Minimize :

$$\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)$$

Subject to :

$$x_1 x_2 \cdots\cdots\cdots x_n = b$$

and $$x_1, x_2, \ldots\ldots, x_n \ge 0$$

9. Prove that every local maximum of the general convex programming problem is its global maximum.

Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. By using bounded variable technique, solve the following linear programming problem :

Maximize :

$$Z = x_1 + 3x_2$$

s.t. $$x_1 + x_2 + x_3 \le 10$$

$$x_1 - 2x_3 \ge 0$$

$$x_2 - x_3 \le 10$$

and $$0 \le x_1 \le 8, \quad 0 \le x_2 \le 4, \quad x_3 \ge 0$$

11. Use branch and bound method to solve the following linear programming problem :

Maximize :

$$Z = 4x_1 + 3x_2$$

Subject to :

$$5x_1 + 3x_2 \ge 30$$

$$x_1 \le 4$$

$$x_2 \le 6,$$

$$x_1, x_2 \ge 0 \text{ and are integers}$$

12. Prove that for each quadratic programming problem :

Maximize :

$$f(x) = C^T X + \frac{1}{2} X^T G X$$

Subject to :

$$AX = b, \quad X \geq 0$$

there exists another quadratic programming problem :

Minimize :

$$L(X, \lambda) = \frac{1}{2} \times X^T G X + \lambda^T b$$

Subject to :

$$-GX + A^T \lambda \geq C$$

$$X \geq 0$$

and λ unrestricted in sign such that if one has a finite optimal solution, then so has the other. Further the optimal values of both the problems are the same.

13. Solve the following quadratic programming problem by using Wolfe's method :

Minimize :

$$f(x_1, x_2) = x_1^2 - x_1 x_2 + 2x_2^2 - x_1 - x_2$$

Subject to :

$$2x_1 + x_2 \leq 1$$

$$x_1, x_2 \geq 0$$