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MA/MSCMT–09

June – Examination 2020

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MA/MSCMT-09

*Time : **3** Hours*] [*Maximum Marks :* ***80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section A contains 8 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 2 marks and maximum word limit may be **30** words. Section

B contains 8 Short Answer Type Questions. Examinees will have to answer any *four* questions. Each question is of 8 marks. Examinees have to delimit each answer in maximum **200** words. Section C contains 4 Long Answer Type Questions. Examinees will have to answer any *two* questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum **500** words. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

1. (i) Define kernel of integral transform.

(ii) Find :

$$L^{-1}\left[\frac{1}{(8p-9)^{2/3}}\right]$$

(iii) Define Fourier sine transform.

(iv) If $M\{f(x); p\} = F(p)$, then prove that :

$$M\{f(a\,x); p\} = a^{-p}\,F(p).$$

(v) Define linear integral equation.

(vi) Define Fredholm integral equation of second kind.

(vii) Define symmetric kernel.

(viii) Define Integro-differential equation.

Section–B **4×8=32**

(Short Answer Type Questions)

2. Prove that :

$$L[E_i(t)]=\frac{\log(p+1)}{p}$$

3. Solve $(D^2 + 1)\,x = t \cos 2t$ given $x(0) = 0$, $x'(0) = 0$.

4. Find inverse Fourier sine transform of $\frac{p}{1+p^2}$.

5. If $\nu > -\frac{1}{2}$, then prove that :

$$H_\nu\{x^{\nu-1}e^{-ax}; p\} = L\{x^\nu J_\nu(px); a\} = \frac{2^\nu p^\nu \Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}(a^2+p^2)^{\nu+\frac{1}{2}}}$$

6. Transform $\frac{d^2y}{dx^2}+xy=1;$ $y(0) = 0$, $y(1) = 1$ into integral equation.

7. Solve the integral equation and find resolvent kernel :

$$g(x) = f(x)+\lambda\int_{-1}^{1}(xt+x^2t^2)\,g(t)\,dt$$

8. Prove that if a kernel is symmetric, then all of its iterated kernels are also symmetric.

9. Solve the integral equation :

$$g(x) = x + \lambda \int_0^1 (4xt - x^2)\, g(t)\, dt$$

Section–C **2×16=32**

(Long Answer Type Questions)

10. Find the inverse Laplace transform of :

(i) $\frac{1}{p} \log \frac{p+2}{p+1}$

(ii) $\frac{1}{(p^2 + a^2)^{3/2}}$

(iii) $\frac{1}{p\sqrt{p+4}}$

(iv) $\log\left(\frac{p + \sqrt{p^2+1}}{2p}\right)$

11. Prove that :

$$M\{e^{-ax} J_\nu(bx); p\} = \frac{b^\nu 2^{p-1}}{\sqrt{\pi}\, \Gamma(\nu+1)}$$

$$(a^2 + b^2)^{-(\nu+p)/2} \times \Gamma\left(\frac{\nu+p}{2}\right) \Gamma\left(\frac{\nu+p+1}{2}\right)$$

$$2F_1\left(\frac{\nu+p}{2}, \frac{\nu-p+1}{2}; \nu+1; \frac{b^2}{a^2+b^2}\right),$$

$$\left(\mathrm{Re}(a) > 0, \nu > -\frac{1}{2}\right)$$

Hence deduce that :

(i) $M\{J_\nu(bx); p\} = \frac{b^{-p} 2^{p-1} \Gamma\left(\frac{\nu+p}{2}\right)}{\Gamma\left(\frac{\nu-p+2}{2}\right)},$

$- \nu < p < \nu + 2$

(ii) $M\{x^{-\nu} J_\nu(x); p\} = \frac{2^{p-\nu-1} \Gamma\left(\frac{p}{2}\right)}{\Gamma\left(\nu - \frac{1}{2} p + 1\right)}, 0 < \mathrm{Re}(p) < 1, \nu > -\frac{1}{2}$

12. Heat is supplied at a constant rate Q per in the plane $z = 0$ to an infinite solid of conductivity K. Show that the steady temperature at a point distance r from the axis of the circular area of radius a and distance z from the plate $r = 0$ is given by :

$$\frac{Qa}{2K}\int_0^{\infty}\left(e^{-pz}\, J_0(pr)\, J_1(ap)\, p^{-1}\right) dp$$

13. (i) Solve :

$$g'(x) = x + \int_0^x g(x-t)\cos t\,dt$$

given $g(0) = 4$.

(ii) Solve :

$$g(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2} g(t)\,dt$$