i.e. f_1, f_2 are r_1-approximate solution and r_2-approximate solution of the equation $\frac{dx}{dt} = g(t, x)$ respectively. Then prove that for all s and t in I

$$\|f_1(t) - f_2(t)\| \le \|f_1(s) - f_2(s)\| e^{c|t-s|} + (r_1 + r_2)\left(\frac{e^{c|t-s|} - 1}{c}\right)$$

Section–C **2×16=32**

(Long Answer Type Questions)

10. State and prove closed graph theorem.

11. If f be a functional defined on a linear subspace M of a normed linear space N, $x_0 \notin$ M and $M_0 = \left[M \bigcup \{x_0\}\right] = \{x + \alpha x_0 : x \in M$, '$\alpha$' is real$\}$ is the linear subspace spanned by M and x_0, then prove that f can be extended to a functional f_0 defined on M_0 such that $\|f_0\| = \|f\|$.

12. If X be a Banach space over the field K of scalars and if $f : [a, b] \to X$ and $g : [a, b] \to R$ be continuous and differentiable functions such that $\| Df(t) \| \le Dg(t)$ at each point $t \in (a, b)$ then prove that $\| f(b) - f(a) \| \le g(b) - g(a)$.

13. State and prove implicit function theorem.

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MA/MSCMT–06

June – Examination 2020

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MA/MSCMT-06

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Section A contains 8 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 2 marks and maximum word limit may be **30** words. Section B contains 8 Short Answer Type Questions. Examinees will have to answer any four 4 questions. Each question is of 8 marks. Examinees have to delimit each answer in maximum **200** words. Section C contains 4 Long Answer Type Questions. Examinees will have to answer any 2 questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum **500** words. Use of non-programmable scientific calculator is allowed in this paper.

Section–A 8×2=16

(Very Short Answer Type Questions)

1. (i) Define complete normed linear space.

(ii) Define closed linear transformation.

(iii) If X be a complex inner product space, then prove that :

$$(\alpha x - \beta y, z) = \alpha(x, z) - \beta(y, z)$$

(iv) Define Orthogonal complement of a set.

(v) Define self-adjoint operator.

(vi) Define perpendicular projection.

(vii) State inverse function theorem.

(viii) Define regulated function.

Section–B 4×8=32

(Short Answer Type Questions)

2. Show that the linear spaces R (real) and C (complex) are normed linear spaces under the norm $|| x || = | x |, x \in$ C. Also show that these spaces are Banach spaces.

3. If N and N′ be normed linear spaces over the same scalar field and let T be a linear transformation of N into N′, then prove that T is bounded if and only if it is continuous.

4. Prove that closed convex subset K of a Hilbert Space H contains a unique vectors of smallest norm.

5. If M and N are closed linear subspaces of Hilbert space H s.t. M $\perp$ N, then prove that the linear subspace M + N is closed.

6. If T_1 and T_2 are normal operators on H with the property that either commutes with adjoint of the other, then prove that $T_1 + T_2$ and $T_1 \cdot T_2$ are normal.

7. If T is normal operator on a Hilbert space H, then prove that eigenspaces of T are pairwise orthogonal.

8. If $[a, b]$ be a compact interval, let g be a regulated function on $[a, b]$ into $\{r \in R : r \geq 0\}$ and if h be a continuous function on $[a, b]$ into R such that for all $t \in [a, b]$

$$h(t) \leq g(t) + c\int_a^t h(s)ds$$

where c is a positive real number, then prove that for all $t \in [a, b]$

$$h(t) \leq g(t) + c\int_a^t g(s)e^{c(t-s)}ds$$

9. Let I be an open interval of R. Let W be an open subset of a real Banach space X and let g be a continuous map of I × W into X such that g is c-lipschitz on W uniformly with respect to I, where c is a positive real number. Let r_1 and r_2 be two positive real numbers such that for all $t \in$ I

$$\| Df_1(t) - g(t, f_1(t)) \| \leq r_1$$

and $$\| Df_2(t) - g(t, f_2(t)) \| \leq r_2$$