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MA/MSC MT–04

December – Examination 2020

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Geometry and Tensors)

Paper : MA/MSC MT-04

*Time : **2** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Section 'A' contains 8 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 2 marks and maximum word limit may be **30** words. Section 'B' contains 8 Short Answer Type Questions. Examinees will have to answer any *four* questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum **200** words. Use of non-programmable scientific calculator is allowed in this paper.

Section–A 8×2=16

(Very Short Answer Type Questions)

1. (i) Define inflexional tangent.
   (ii) Define Binormal.
   (iii) Define Indicatrix.
   (iv) Define ruled surface.
   (v) Define Trajectory.
   (vi) State Meunier's theorem.
   (vii) State Gauss-Bonnet theorem.
   (viii) Define covariant tensor of second order.

Section–B 4×16=64

(Short Answer Type Questions)

2. Find the equation of osculating plane at a point of a space curve given by the intersection of two surfaces.
3. Prove that the indicatrix at a point of the surface $z = f(x, y)$ is a rectangular hyperbola if $(1 + p^2)t + (1 + q^2)r - 2pqs = 0$
4. Prove that the metric of a surface is invariant under parametric transformation.

5. Prove that there are two principal directions at every point on a surface which are mutually orthogonal.
6. Prove that the geodesic curvature vector of any curve is orthogonal to the curve.
7. A covariant tensor of first order has components xy, $2y - z^2$, xz in rectangular coordinates. Determine its covariant components in spherical polar coordinates.
8. If A^{ijk} is a skew-symmetric tensor, show that $\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^k}\left(\sqrt{g}A^{ijk}\right)$ is a tensor.
9. Prove that :

$$R^{\alpha}_{.ijk,p} + R^{\alpha}_{.ikp,j} + R^{\alpha}_{.ipj,k} = 0$$