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MA/MSCMT–03

December – Examination 2020

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variations and Special Function)

Paper : MA/MSCMT-03

*Time : **2** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A **8×2=16**

Note :– Answer all questions. Each question carries 2 marks.

1. (i) Write the Monge's subsidiary equations for $x^2r + 2xys + y^2t = 0$.

(ii) Write down the Riccati's equation.

(iii) The PDE $\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = 0$ is parabolic. (True/False)

(iv) The singular points of the differential equation $x(x-1)^2y'' + 2xy' + (x-1)y = 0$ are .... and ....

(v) $\lim_{b \to 0} \{ {}_2F_1(1, b; 1; z/b) \} = ...$

(vi) What is value of $P_1(x)$?

(vii) Define Eigenfunction.

(viii) Write down Euler-Lagrange equation.

Section–B **4×16=64**

Note :– Answer any *four* questions. Each question carries 16 marks.

2. Solve :

$$y^3 \frac{d^2y}{dx^2} = C$$

3. Find the system of curves satisfying the differential equation :

$$xdx + ydy + c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}dz = 0$$

which lie on the surface :

$$1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

4. Solve :

$$x - t = \frac{x}{y^2}$$

5. Find the characteristics of :

$$x^2r + 2xys + y^2t = 0$$

6. Solve the two-dimensional Heat conduction equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial u}{\partial t}$ by the method of separation of variable.

7. Find the curve with fixed boundary revolves such that its rotation about x-axis generate minimal surface area.

8. If the complete elliptic integral of first kind being :

$$K = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}$$

Show that :

$$K = \frac{\pi}{2} \, {}_2F_1\left[\frac{1}{2}; \frac{1}{2}; 1, k^2\right]$$

9. Prove that :

$(2n + 1)(1 - x^2) Q'_n = n(n + 1)(Q_{n-1} - Q_{n+1})$