731

# MA/MSCMT–02

## December – Examination 2020

# M.A./M.Sc. (Previous) Examination

## MATHEMATICS

## Real Analysis and Topology

## Paper : MA/MSCMT-02

*Time : **2** Hours* ] [ *Maximum Marks : **80***

***Note*** :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

**Section–A** **8×2=16**

**(Very Short Answer Type Questions)**

***Note*** :– Answer all questions. As per the nature of the questions delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Measurable Set.

(ii) Define Measurable Function.

(iii) State Holder's Inequality.

(iv) State Riesz-Fisher theorem.

(v) Write the necessary and sufficient conditions for a bounded function $f$ defined on the interval $[a, b]$, to be L-integrable.

(vi) State Parseval's identity.

(vii) Define Topological Space.

(viii) Define Compact Topological Space.

**Section–B** **4×16=64**

**(Short Answer Type Questions)**

***Note*** :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Let $\{E_n\}$ be a countable collection of sets of real numbers, then show that :

$$m^*\left(\bigcup_n E_n\right) \leq \sum_n m^*(E_n)$$



3. Let $f$ be a measurable function finite on $E = [a, b]$. Then prove that for given $\in > 0$, there exists a function $\varphi$, continuous on $[a, b]$ such that

$$m(\{x \in E : f(x) \neq \varphi(x)\}) < \in.$$

4. Show that every bounded measurable function $f$ defined on a measurable set E is L-integrable.

5. State and prove Minkowski's inequality.

6. Prove that for a subset A of a topological space $(X, \tau)$, $\bar{A} = A \cup A'$.

7. Show that the property of a space being a Hausdorff space is a hereditary property.

8. Prove that closure of a connected set is connected.

9. Prove that every open continuous image of a locally compact space is locally compact.