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MA/MSCMT–01

December – Examination 2020

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Advanced Algebra)

Paper : MA/MSCMT-01

*Time : **2** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer up to **30** words. Each question carries 2 marks.

1. (i) Define direct product of groups.

   (ii) Define derived subgroup.

(iii) Define Euclidean ring.

(iv) Define cyclic module.

(v) Define nullity of a linear map.

(vi) Define algebraic field extension.

(vii) Define inner product space.

(viii) Define self-adjoint linear maps.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Delimit your answer upto **200** words. Each question carries 16 marks.

2. Let G be a group and suppose that G is the internal direct product of its subgroups H_1, H_2 ..., H_n. Let :

$$G^* = H_1 \times H_2 \times .... \times H_n$$

Then show that G and G^* are isomorphic.

3. Show that any *two* conjugate class C[*a*] and C[*b*] of a group G are either disjoint or identical.

4. Let M be an R-module and let N be a sub-module of M. Show that the additive abelian quotient group M/N can be made into an R-module by defining scalar multiplication as :

$$r(N + x) = N + rx \ \forall \ r \in R, N + x \in M/N$$

5. Let K/F be a field extension and let $a \in K$ be algebraic over F. Then show that any two minimal monic polynomials for a over F are equal.

6. Let H be a subgroup of all automorphisms of field K. Then show that the fixed field of H is a subfield of K.

7. Show that a linear transformation $t : V \to V$ is invertible if and only if matrix of t relative to some basis B of V is invertible.

8. Let V be an inner product space. Show that for any arbitrary vectors

$$u, v \in V \ | <u, v> | \leq || u || \ || v ||.$$

9. Show that any *two* finite fields having same number of elements are isomorphic.