7. Show that function $g(x)=\sin\left(\frac{\pi x}{2}\right)$ is a solution of integral equation :

$$g(x)-\frac{\pi^2}{4}\int_0^1 K(x,t)\,g(t)\,dt=\frac{x}{2}$$

where :

$$K(x,t)=\begin{cases}\frac{x(2-t)}{2}, & 0\le x\le t\\ \frac{t(2-x)}{2}, & t\le x\le 1\end{cases}$$

8. Prove that the characteristic numbers of a symmetric kernel are real.

9. Solve :

$$g(x)=x+\lambda\int_0^1 (xt^2+x^2t)\,g(t)\,dt$$

MA/MSCMT–09

December – Examination 2020

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MA/MSCMT-09

*Time : **2** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B.

Section A contains eight Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 2 marks and maximum word limit may be **30** words. Section B contains eight Short Answer Type Questions. Examinees will have to answer any *four* questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum **200** words.

Section–A 8×2=16

(Very Short Answer Type Questions)

1. (i) If $L(f(t)) = F(p)$, then prove that :

$$L(f(at)) = \frac{1}{a} F\left(\frac{p}{a}\right)$$

(ii) Evaluate :

$$L^{-1}\left\{\frac{5}{(p-2)^2+25}\right\}$$

(iii) Write relationship between Fourier Transform and Laplace Transform.

(iv) If $M\{f(x)\} = F(p)$, then prove that :

$$M\{x^a f(x)\} = F(p + a)$$

(v) Define Hankel Transform.

(vi) Define Volterra integral equation.

(vii) Define Abel's Integral Equation.

(viii) Define Symmetric Kernel.

Section–B 4×16=64

(Short Answer Type Questions)

2. Evaluate :

$$L\left\{\frac{1-\cos t}{t^2}\right\}$$

3. Solve $ty'' + (t - 1)y' - y = 0$, given $y(0) = 5$, $y(\infty) = 0$.

4. Find Fourier sine and cosine transform of :

$$f(t) = \begin{cases} t & 0 < t < 1 \\ 2-t & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

5. Prove that :

$$M\{x^{\rho}(1-x)^{c-1}\ {}_2F_1(a,b;c;1-x)\,H(1-x);p\} = \frac{\Gamma(c)\Gamma(p+\rho)\,\Gamma(p-a-b+c+\rho)}{\Gamma(p-a+c+\rho)\,\Gamma(p-b+c+\rho)}$$

6. Prove that :

$$H_{\nu}\{x^{\nu}(a^2-x^2)^{\mu-\nu-1}U(a-x);p\} = 2^{\mu-\nu-1}\Gamma(\mu-\nu)p^{\nu-\mu}a^{\mu}J_{\mu}(pa)\ a>0, \mu>\nu>0$$