MAMT-09/MSCMT-09/MAT-103

December - Examination 2025
M.A./M.Sc. (Final) Examination
MATHEMATICS
(INTEGRAL TRANSFORMS AND INTEGRAL EQUATION)
Paper : MAMT-09/MSCMT-09/MAT-103

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** the questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Find Laplace Transform sin(at) sin(bt).

   (ii) Find Inverse Laplace Transform of $\frac{1}{(p^2-2p+5)}$.

   (iii) What is one dimensional heat conduction equation?

   (iv) Define Fourier cosine transform.

   (v) Find $M[e^{-ax}; p]$.

   (vi) Define Hankel Transform.

   (vii) Define Fredholm Integral equation.

   (viii) Define Degenerate Kernel.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. Each answer should not exceed **200** words. Each question carries **8** marks.

2. Show that $L[f(t); p]$ is $\left(\frac{1}{p^2}+\frac{1}{p}\right)e^{-p}-\left(\frac{1}{p^2}+\frac{2}{p}\right)e^{-2p}$ where

$$f(t)=\begin{cases}0 & ; \quad 0<t<1\\ t & ; \quad 1<t<2\\ 0 & ; \quad t>2\end{cases}$$

3. Evaluate $\int_0^t \sin u \cos(t-u)\, du$.

4. Find Fourier sine and cosine transform of $f(t) = \begin{cases} t & ; \ 0 < t < 1 \\ 2-t & ; \ 1 < t < 2 \\ 0 & ; \ t > 2 \end{cases}$

5. Prove that $M\{(1+x)^{-a}; p\} = \frac{\Gamma(p)\Gamma(a-p)}{\Gamma(a)}$, $0 < Re(p) < Re(a)$.

6. Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation -

$$g(x) = \sin x + 2\int_0^x \cos(x-t)g(t)dt$$

7. Transform $\frac{d^2y}{dx^2} + xy = 1; y(0) = 0, y(1) = 1$ into an integral equation.

8. Solve $g(x) = e^x + \lambda \int_0^1 2e^x e^t g(t)dt$.

9. Solve by the method by successive approximation -

$$g(x) = \frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2} + \frac{1}{2}\int_0^1 tg(t)dt$$

Section-C 2×16=32

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. (a) Solve : $(D^2+1)y = t\cos 2t,\ y(0) = 0, \left(\frac{dy}{dt}\right)_{t=0} = 0$.

(b) Solve BVP - $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$; $x > 0, t > 0$ with boundary conditions

$u(x, 0) = 0$; $u_t(x, 0) = 0$, $x > 0$

$u(0, t) = f(t)$, $\lim_{x\to\infty} u(x, t) = 0$, $t \geqslant 0$

11. Find $D(\lambda)$ and $D(x, t; \lambda)$ & solve the equation -

$$g(x) = x + \lambda \int_0^1 \left(x^t + \sqrt{xt}\right) g(t)dt.$$

12. Solve with the help of Hilbert-Schmidt theorem -

$$g(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^1 (xt + x^2t^2)\, g(t)dt$$

13. Find the resolvent kernel of the following integral equation -

$$g(x) = 1 + \lambda \int_0^1 (1 - 3xt)\, g(t)dt$$

for what value of λ, the solution does not exist. Also find the solution of above integral equation.
