MAMT-03/MSCMT-03/MAT-201

December - Examination 2025
M.A./M.Sc. (Previous) Examination
MATHEMATICS
DIFFERENTIAL EQUATIONS, CALCULUS OF VARIATION AND SPECIAL FUNCTIONS
Paper : MAMT-03/MSCMT-03/MAT-201

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** the questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) What is the general form of a second-order PDE in two variables?

 (ii) Write the Monge's subsidiary equations.

 (iii) Write the three-dimensional Laplace equation in spherical coordinates.

 (iv) Find the nature of following PDE -

$$3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\,\partial y} + 5\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial y} = 0$$

 (v) Define functional.

 (vi) Write generating function for Hermite polynomial.

 (vii) Write Rodrigues formula for $L_n(x)$.

 (viii) Write orthogonal property for Legendre polynomial.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. Each answer should not exceed **200** words. Each question carries **8** marks.

2. Find the general solution of the Riccati's equation -

$$\frac{dy}{dx} = 2 - 2y + y^2$$

Whose one particular solution is $(1 + \tan x)$.

3. Solve - $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})\,dz = 0$.

4. Find the characteristics of $y^2r - x^2t = 0$.

5. Solve the following Sturm-Liouville problem

$$y'' + \lambda y = 0;\ y'(-\pi) = 0,\ y'(\pi) = 0.$$

6. Find the extremals of the functional

$$F[y(x)] = \int_{x_1}^{x_2} \frac{(1+y'^2)^{1/2}}{x} dx.$$

7. Prove that - $B(\lambda, c - \lambda)\ {}_2F_1(a, b; c; z) = \int_0^1 t^{\lambda-1}(1-t)^{c-\lambda-1}\ {}_2F_1(a, b; c; zt)dt$, where $|z| < 1$, $\lambda > 0$, $c - \lambda > 0$.

8. Show that - $(2n+1)(1-x^2)Q_n' = n(n+1)(Q_{n-1} - Q_{n+1})$

9. State and prove Rodrigues formula for Hermite polynomial.

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. (i) Solve the two-dimensional wave equation using the method of separation of variables.

(ii) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

11. State and prove Euler-Lagrange's equation.

12. (i) Define Gauss's Hypergeometric series and discuss its convergence conditions.

(ii) Show that -

$$\int_0^{\pi} x^2 P_{n+1} P_{n-1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

13. State and prove orthogonal property of Bessel function.
