MSCMT- 04/MAMT- 04

December - Examination 2025
M.A./M.Sc. (Previous) Examination
MATHEMATICS
DIFFERENTIAL GEOMETRY AND TENSOR
Paper : MSCMT-04/MAMT-04

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** the questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define Torsion.
   (ii) Define osculating circle.
   (iii) Define Bertrand curves.
   (iv) Show that the surface $z - c = \sqrt{xy}$ is developable.
   (v) Define lines of curvature.
   (vi) For Monge's form of surface $z = f(x, y)$ write the equation of asymptotic line.
   (vii) Write geodesic curvature in terms of normal angle $\overline{w}$.
   (viii) What are the Weingarten formulae?

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. Each answer should not exceed **200** words. Each question carries **8** marks.

2. Show that the osculating plane at (x, y, z) on the curve $x^2 + 2ax = y^2 + 2by = z^2 + 2cz$ has the equation:

$$(b^2 - c^2)(x + a)^2 (X - x) + (c^2 - a^2)(y + b)^2 (Y - y) + (a^2 - b^2)(z + c)^2 (Z - z) = 0$$

3. Prove that a curve is uniquely determined, expect as to position in space, when its curvature and torsion are given functions of its arc-length.
4. Show that on a right helicoid, the family of curves orthogonal to the curves $u \cos \nu$ = constant is the family $(u^2 + a^2)\sin^2 v$ = constant.
5. Find the equation of the right conoid generated by lines which meet OZ, are parallel to the plane XOY and intersect the circle $x = a$, $y^2 + z^2 = r^2$.

6. Show that the directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$, are conjugate if $LR - 2MQ + NP = 0$.
7. Show that the metric of a Euclidean space, referred to cylindrical coordinates is given by –

   $$ds^2 = (dr)^2 + (r\, d\theta)^2 + (dz)^2$$
8. Contract the Riemann Christoffel tensor and find Ricci Tensor.
9. Prove that: $A^{ij}_{j} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^j}\left(A^{ij}\sqrt{g}\right) - A^{jk}\begin{Bmatrix} i \\ jk \end{Bmatrix}$. Show that last term vanishes if A^{ij} is skew symmetric.

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Find the inflexional tangent at (x_1, y_1, z_1), on the surface $y^2z = 4ax$.
11. Explain geodesic and derive geodesic on surface of revolution given by:

    $x = u\cos\theta,\ y = u\sin\theta,\ z = f(u)$.
12. State and prove Gauss-Bonnet theorem.
13. State and prove Ricci's theorem.
