MAMT-06/MSCMT-06

December - Examination 2025

M.A./M.Sc. (Final) Examination

MATHEMATICS

ANALYSIS AND ADVANCED CALCULUS

Paper : MAMT-06/MSCMT-06

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define convergence in normed linear space.

(ii) What is multi linear mapping?

(iii) State open mapping theorem.

(iv) Define closed linear transformation.

(v) Write Bessel's inequality for finite orthonormal sets.

(vi) Define homeomorphism for Banach space X and Y.

(vii) State polarization identity in a Hilbert Space.

(viii) Define Lipschitz's property.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. Each answer should not exceed **200** words. Each question carries **8** marks.

2. State and prove Minkowski's inequality for C^n.

3. If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exist a functional F in conjugate space N* such that $F(M) = \{0\}$ and $F(x_0) \neq 0$.

4. Prove that a closed convex subset K of a Hilbert space H contains a unique vector of smallest norm.

5. Show that the set of unitary operators on a Hilbert space H, forms a multiplicative group.

6. State and prove mean value theorem for Banach space.

7. Let f be a regulated function on a compact interval [a, b] of R into R such that a < b and for all t in [a, b], f(t) ≥ 0. Then prove that $\int_a^b f(t)dt \geq 0$. Further prove that if f be a continuous function at a point c of [a, b] and f(c) > 0, then $\int_a^b f(t)dt > 0$.

8. If T is normal operator on a Hilbert space H, then prove that eigenspaces of T are pairwise orthogonal.

9. Let f be a C^1 map on a compact interval [a, b] into a compact interval [c, d] of R and let g be a continuous function on [c, d] into a Banach space x over k, then prove that

$$\int_a^b \left(D f(s) g\left(f(s)\right)\right) ds = \int_{f(a)}^{f(b)} g(t)dt$$

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. State and prove natural embedding theorem for normed linear space.

11. If B and B' be Banach spaces and T a continuous linear transformation of B on to B', then prove that the image of every open sphere centred at origin in B contains an open sphere centred at origin in B'.

12. If N be a normed linear space and x_0 is a non-zero vector in N, then ∃ a continuous linear functional F defined on the conjugate space N* s.t. $f(x_0) = \|x_0\|$ and $\|F\| = 1$ prove it.

13. State and prove spectral theorem.
