MAMT-02/MSCMT-02

December - Examination 2025

M.A./M.Sc. (Previous) Examination

MATHEMATICS

REAL ANALYSIS AND TOPOLOGY

Paper : MAMT-02/MSCMT-02

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** the questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define measurable set.

   (ii) Define Lebesgue measure of a set.

   (iii) Define orthonormal system.

   (iv) State Riesz-Fischer theorem.

   (v) Write the necessary and sufficient conditions for a bounded function f defined on the interval [a, b], to be L-integrable.

   (vi) State Parseval's identity.

   (vii) Define Topological space.

   (viii) Is a finite topological space compact?

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. Each answer should not exceed **200** words. Each question carries **8** marks.

2. Show that intersection of two measurable sets is also a measurable set.

3. Let f be a measurable function finite on E = [a, b]. Then prove that for given $\in > 0$, there exists a function φ, continuous on [a, b] such that $m(\{x \in E : f(x) \neq \varphi(x)\}) < \in$.

4. If the function f and g are Lebesgue integrable over the measurable set E and if $f(x) < g(x)$ on E, then prove that

$$\int_E f(x)dx \leq \int_E g(x)dx$$

5. State and prove Minkowski's inequality.
6. Prove that homeomorphism is an equivalence relation in the family of topological spaces.
7. Prove that a closed subspace of normal space is a normal space.
8. Show that every metric space is a T_2-space.
9. Show that a function $f: X \to Y$ is continuous iff the inverse of each member of a base **B** for Y is an open subset of X.

Section-C

2×16=32

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. State and prove Weierstrass Approximation Theorem.
11. Prove that L_2 is a complete space.
12. (i) If E is a countable set, then show that $m^*(E) = 0$.

    (ii) Prove that a subset A of Y is closed in the quotient topology τ_f relative to $f: X \to Y$ iff $f^{-1}(A)$ is closed in X.
13. Show that a topological X is disconnected iff there exists a proper subset of X which is both open and closed in X.
