MSCMT-01/MAMT-01

December - Examination 2025

M.A./M.Sc. (Previous) Examination

MATHEMATICS

ADVANCED ALGEBRA

Paper : MSCMT-01/MAMT-01

[*Time: **3** Hours*] [*Maximum Marks: **80***]

Note :- The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section-A **8×2=16**

(Very Short Answer Type Questions)

Note :- Answer **all** the questions. As per the nature of the question, delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries **2** marks.

1. (i) Define internal direct product.
   (ii) Define solvable group.
   (iii) Define units in commutative ring with unity.
   (iv) Define algebraic field extension.
   (v) Define nullity of a matrix.
   (vi) Define minimal polynomial of a matrix.
   (vii) State Pythagoras Theorem.
   (viii) Define orthogonal linear transformation.

Section-B **4×8=32**

(Short Answer Type Questions)

Note :- Answer **any four** questions. Each answer should not exceed **200** words. Each question carries **8** marks.

2. Show that any two conjugate classes C[a] and C[b] of a group G are either disjoint or identical.
3. Let G be a group and H be a subgroup of G. If $H \lhd G$ and G/H is Abelian then show that $G' \subseteq H$. Conversely if $G' \subseteq H$, then $H \lhd G$ and G/H is Abelian, where G' denotes derived subgroup.
4. Show that a finite group G is solvable if and only if G has a composition series whose factors are cyclic of prime order.
5. Let R be a Euclidean ring and a, b be any two non-zero elements in R. If b is not a unit in R, then show that $d(a) < d(ab)$.
6. Let R be a ring with unity and M be an R-module. Let N be finitely generated submodule of M generated by a subset $A = \{a_1, a_2, \ldots, a_n\}$ of M.
   Then show that $N = RA = Ra_1 + Ra_2 + \ldots\ldots\ldots\ldots + Ra_n$.

7. If F is a field, then show that every polynomial $f(x) \in F[x]$ has a splitting field.
8. Show that every orthonormal set of vectors is a linearly independent set in an inner product space.
9. Show that the eigenvalues of a self-adjoint linear transformation are real.

Section-C **2×16=32**

(Long Answer Type Questions)

Note :- Answer **any two** questions. You have to delimit your each answer maximum up to **500** words. Each question carries **16** marks.

10. Show that every group is isomorphic to a group of permutations.
11. Show that a group is solvable if and only if $G^{(n)} = \{e\}$ for some $n \in N$.
12. If K is a finite extension of a field F, then show that the group G(K/F) of F-automorphisms of K is finite and $O[G(K/F)] \leq [K:F]$.
13. Show that any two finite dimensional vector spaces over the same field are isomorphic if and only if they are of the same dimension.
