MAMT–10/MSCMT–10

December – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Mathematical Programming)

Paper : MAMT–10/MSCMT–10

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define convex function.

(ii) Define Slack and surplus variable.

(iii) Write Sylvester's law for definiteness of matrices.

(iv) Write necessary condition for the function $f(x, \lambda)$ have a saddle point on (x_0, λ_0).

(v) State Bellman's principle of Optimality.

(vi) Define transition function in Dynamic programming.

(vii) Consider the following problem :

Minimize $z = f(X)$,

Subject to $g_j(X) \leq 0$; $j = 1, 2, 3, \ldots\ldots\ldots, m$.

Then write the suitable Kuhn-Tucker conditions.

(viii) Write quadratic form :

$$Q(x) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$$

in matrix form.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that the sum of convex function is convex and if at least one of the function is strictly convex then prove that the sum is strictly convex.

3. Solve the following integer programming problem by branch and bound algorithm.

   Min $z = x_1 + x_2$

   Subject to $3x_1 + 2x_2 \le 12$

   $$x_2 \le 2$$

   $x_1, x_2 \ge 0$ and integers.

4. Find the dimensions of a rectangular parallelopiped with largest volume to be inscribed in ellipsoid.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

5. Solve the following programming problem graphically :

   Minimize $f(x_1, x_2) = x_1^2 + x_2^2$

   Subject to

   $$x_1 + x_2 \ge 4$$

   $$2x_1 + x_2 \ge 5$$

   $$x_1, x_2 \ge 0.$$

6. Use Lagrangian multiplier method to solve the following nonlinear programming problem :

   Min $f(x_1, x_2) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 24x_1 - 8x_2 - 12x_3 + 10 + \lambda(x_1 + x_2 + x_3 - 11)$

   Subject to

   $$x_1 + x_2 + x_3 = 11$$

   $$x_1, x_2, x_3 \ge 0.$$

7. Prove that every local maximum of the general convex programming problem is its global maximum.

8. Divide a quantity 'b' into n parts in such a way that their product is maximum.

9. Write the Kuhn-Tucker necessary and sufficient conditions for the following non-linear programming problem to have on optional solution :

Min $f(x_1, x_2) = x_1^2 - 2x_1 - x_2$

Subject to

$$2x_1 + 3x_2 \le 6$$
$$2x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0.$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Solve the following linear programming problem by revised simplex method :

Max $z = 3x_1 + 6x_2 + 2x_3$

Subject to

$$3x_1 + 4x_2 + x_3 \le 2$$
$$x_1 + 3x_2 + 2x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0.$$

11. Solve the following quadratic programming using Wolfe's method :

Min $f(x_1, x_2) = x_1^2 + 2x_2^2 + 2x_3^2 - 8x_1 - 10x_2$

Subject to

$$x_1 + x_2 \le 5$$
$$x_1 + 2x_2 \le 8$$
$$x_1, x_2 \ge 0$$

and are integers.

12. A manufacturer of baby doll makes two types of dolls doll x and doll y. Processing of these two dolls is done on two machine, A and B. Doll x requires 2 hours on machine A and 6 hours on machine B. Doll y requires 5 hours on machine A and also 5 hours on machine B. There are sixteen hours of time per day available on machine A and thirty hours on machine B. The profit gained on both the dolls is same. *i.e.*, one rupee per doll. What should be the daily production of the two dolls for maximum profit ?

(i) Set up and solve the LPP.

(ii) If optimal solution is not integer valued, use Gomory's technique to derive the optimal solution.

13. Use dynamic programming to solve the following L.P.P. :

Max $z = 2x_1 + 5x_2$

Subject to

$$2x_1 + x_2 \leq 43$$

$$2x_2 \leq 46$$

$$x_1, x_2 \geq 0.$$