MAMT–09/MSCMT–09

December – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MAMT–09/MSCMT–09

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define piecewise continuous function.

(ii) Define periodic function and give an example.

(iii) If :

$$L^{-1}\left\{\frac{p^2-1}{(p^2+1)^2}\right\}=t\cos t$$

then find :

$$L^{-1}\left\{\frac{9p^2-1}{(9p^2+1)^2}\right\}$$

(iv) Define Fourier cosine transform.

(v) State the Mellin inversion theorem.

(vi) Volterra integral equation.

(vii) Define singular integral equation.

(viii) Define symmetric kernels.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find Laplace transform of :

$$\frac{1-\cos t}{t^2}$$

3. Solve :

$$(D^2 + 1)y = t \cos 2t,\ y(0) = 0,\ y'(0) = 0$$

4. Find the Fourier transform $f(t)$, where :

$$f(t) = \begin{cases} 1-t^2 & , \ |t|<1 \\ 0 & , \ |t|>1 \end{cases}$$

and hence evaluate :

$$\int_0^\infty \left(\frac{t\cos t - \sin t}{t^3}\right)\cos\frac{t}{2}\,dt$$

5. Prove that :

$$M\{x^{\rho}(1-x)^{c-1}\,{}_2F_1(a, b; c; 1-x)\,H(1-x); p\}$$

$$= \frac{\Gamma(c)\Gamma(p+\rho)\Gamma(p-a-b+c+\rho)}{\Gamma(p-a+c+\rho)\Gamma(p-b+c+\rho)}$$

6. Show that the characteristic numbers of a symmetric kernel are real.

7. Find the resolvent kernel of the Volterra integral equation with the kernel :

$$K(x,t) = \frac{(2+\cos x)}{(2+\cos t)}$$

8. Using Hilbert schmidt theorem, find the solution of the symmetric integral equation :

$$g(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^{1}(xt + x^2t^2)\,g(t)\,dt$$

9. Solve the following integral equation :

$$g(x) = x + \lambda \int_0^1 \left(4xt - x^2\right) . g(t)\, dt$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Find the inverse Laplace transform of :

(i) $\dfrac{p}{(p^2 + a^2)^2}$

(ii) $\dfrac{p+1}{(p^2 + 2p + 2)^2}$

(iii) $\log\left(1 + \dfrac{1}{p^2}\right) \text{or} \log\left(\dfrac{p^2+1}{p^2}\right)$

(iv) $\cot^{-1}(p - 1)$

11. State and prove Inversion formula for the Hankel transform.

12. Solve :

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}, x > 0, t > 0$$

If :

(i) $V_x(0, t) = 0$

(ii) $V(x, 0) = \begin{cases} x & , \quad 0 \le x \le 1 \\ 0 & , \quad x > 1 \end{cases}$

(iii) $V(x, t)$ is bounded.

13. How that the integral equation :

$$g(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t).g(t)dt$$

possesses no solution for $f(x) = x$ but that it possesses infinitely many solution when $f(x) = 1$.