9. Prove that :

$$\int_0^{\infty} e^{-st} L_n(t)dt = \frac{1}{s}\left(1-\frac{1}{s}\right)^n.$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Find the eigenvalues and eigenfunctions for the following boundary value problem :

$$y'' - 4y' + (4 - 9\beta)y = 0$$
$$y(0) = 0$$
$$y(a) = 0$$

where '*a*' is a positive real constant.

11. Obtain the surface of minimum area, stretched over a given closed curve C, enclosing the domain D in the *xy*-plane.

12. State and prove Euler-Lagrange Equation.

13. (i) State and prove Rodrigues formula for Hermite Polynomial.

(ii) Prove that :

$$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$

MAMT–03/MSCMT–03

December – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variations and Special Functions)

Paper : MAMT–03/MSCMT–03

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Write Rodrogue's formula for the Laguerre polynomial.

   (ii) Write Monge's subsidiary equations.

   (iii) Classify the following partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial y^2} = 4\frac{\partial^2 z}{\partial x \partial y}$$

   (iv) Write two-dimenstional Laplace equation in Polar cooridnate system.

   (v) Check whether the following boundary value problems are Sturm-Liouvill problem or not :

$$e^x y'' + e^x y' + \beta y = 0$$
$$y(0) = 0$$
$$y'(1) = 0$$

   (vi) Write Monge's subsidiary equations for :

$$yr + s(x - y) - tx + q - p = 0.$$

   (vii) Write Bessal's Function of first kind of index n.

   (viii) Write generating function for Hermite Polynomial.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find the general solution of the Riccati's equation $\frac{dy}{dx} = 2 - 2y + y^2$ whose one particular solution is $(1 + \tan x)$.

3. Solve :

$$(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0$$

4. Solve the following Sturm-Liouville problem :

$$y'' + \beta y = 0$$
$$y'(-\pi) = 0$$
$$y'(\pi) = 0$$

5. Find the series solution of the linear differential equation :

$$4xy'' + 2y' + y = 0$$

6. Find extremals of the functional :

$$F[y(x)] = \int_0^1 \left[y'^2 + x^2 \right] dx;$$
$$y(0) = 1, y(1) = 2$$

7. Derive integral representation of hypergeometric function.

8. Discuss the relation between $J_n(x)$ and $J_{-n}(x)$, n being and integer.