MAMT–02/MSCMT–02

December – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Real Analysis and Topology)

Paper : MAMT-02/MSCMT-02

*Time : **3** Hours*]　　　　[*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A　　　　**8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

Section–C　　　　**2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. State and prove Weierstrass approximation theorem.

11. Prove that L_2 is a complete space.

12. (i) If E is a countable set, then show that $m^*(E) = 0$.

    (ii) Prove that a subset A of Y is closed in the quotient topology τ_f relative to $f : X \to Y$ iff $f^{-1}(A)$ is closed in X.

13. Prove that union of arbitrary family of connected subset of a topological space is connected if the family is with non-empty intersection.

1. (i) Define measurable set.

   (ii) What is the significance of reflexivity in a directed set ?

   (iii) Define orthonormal system.

   (iv) State Weierstrass approximation theorem.

   (v) State Holder's inequality.

   (vi) What do you mean by complete orthonormal system ?

   (vii) Define compact topological space.

   (viii) Define subbase of a filter.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Let E is a measurable set, then for any set A show that :

$$m^*(E \cup A) + m^*(E \cap A) \leq m^*(E) + m^*(A)$$

3. Let f be a measurable function finite on $E = [a, b]$. Then prove that for given $\in > 0$, there exists a function φ, continuous on $[a, b]$ such that :

$$m(\{x \in E : f(x) \neq \varphi(x)\}) < \in$$

4. If the function f and g are Lebesgue integrable over the measurable set E and if $f(x) < g(x)$ on E, then prove that :

$$\int_E f(x)dx \leq \int_E g(x)dx$$

5. State and prove Minkowski's inequality.

6. Prove that for a subset A of a topological space :

$$(X, \tau), \bar{A} = A \cup A'$$

7. Show that the property of a space being a Haudorff space is a hereditary property.

8. Show that a function $f : X \to Y$ is continuous iff the inverse of each member of a base B for Y is an open subset of X.

9. Prove that every filter F on a non-empty set X is contained in an ultrafilter on X.