Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. If H and K are subgroups of G with K normal in G, then show that $H \cap K$ is a normal subgroup of H and $\frac{HK}{K} \cong \frac{H}{(H \cap K)}$.

11. If $F \subset K \subset E$ are fields with [E : K] and [K : F] are finite, then show that E/F finite extension and

$$[E : F] = [E : K]\,[K : F]$$

12. Let F be a field and $f(x) \in F[x]$ be a polynomial having n distinct roots in the splitting field K, then show that Galois group G(K/F) is isomorphic to a subgroup of the symmertric group S_n, and so its order is a divisor of $n!$.

13. State and prove Bessel's inequality.

MAMT-01/MSCMT-01

December – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Advanced Algebra)

Paper : MAMT-01/MSCMT-01

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A 8×2=16

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question, delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define internal direct product.

(ii) Define solvable group.

(iii) Define prime element.

(iv) Define dual basis.

(v) Define splitting field.

(vi) Define Eigen Vector

(vii) Define orthonormal set.

(viii) Define self adjoint linear map.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Show that a group G is abelian if and only if G' = $\{e\}$, e being the identity of G.

3. If a gourp G has a solvable homomorphic image whose kernel is solvable, then show that the group is solvable.

4. If M_1 and M_2 are submodule of an R-module M, then prove that :

   $M_1 + M_2 = \{m_1 + m_2 | m_1 \in M_1, m_2 \in M_2\}$ is a submodule of M.

5. If V is an finite dimensional vector space over a field F, then prove that for every non-zero vector $v \in$ V, there exist a linear functional f in V* such that $f(v) \neq 0$.

6. If a square matrix of order n, over a field F, has n distinct eigen values $\lambda_1, \lambda_2......, \lambda_n$, then prove that there is an invertible matrix P such that P^{-1} A P = drag $(\lambda_1, \lambda_2, ..., \lambda_n)$.

7. State and prove Schwartz's inequality.

8. Let V and V′ be inner product spaces. Then show that a linear transformation $t : V \to V'$ is orthogonal if and only if $\| t(u) \| = \| u \| \ \forall u \in V$.

9. Let V be an innerproduct space, and $A = \{v_i\}^n_{i=1}$ be an orthonormal set in V. Then show that for any vector $v \in$ V, the vector $u = v - \sum_{i=1}^{n} v_i < v, v_i >$ is orthogonal to each v_j, $j = 1, 2, ...n$.