8. If x and y are any two vectors in an inner product space X, then show that :

$$|(x, y)| \leq \|x\| \|y\|$$

9. State and prove global uniqueness theorem.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. State and prove Natural embedding theorem for normed linear space.

11. If B and B′ be Banach spaces and T is a continuous linear transformation of B into B′, then prove that T is continuous if and only if its graph is closed.

12. State and prove spectral theorem.

13. State and prove implicit function theorem on differentiable functions over Banach space.

MAMT–06/MSCMT–06

December – Examination 2023

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MAMT–06/MSCMT–06

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define bounded linear transformation for normed vector space.

   (ii) Define Dual space.

   (iii) State Pythagorean theorem.

   (iv) Define Eigenvalue and Eigenvector of an operator.

   (v) Define Ortho-normal set.

   (vi) State Lipchitz's function in a Banach space.

   (vii) Define Regulated function.

   (viii) Define the graph of a function.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that every normed linear space is a metric space.

3. If M be a closed linear subsapce of a normed linear space N and x_0 is a vector not in M, then prove that $\exists$ a functional F in conjugate space N* s.t. $F(M) = \{0\}$ and $F(x_0) \neq 0$.

4. Show that the set of unitary operators on a Hilbert space H, forms a multiplicative group.

5. State and prove Bessel's inequality in Hilbert Spaces.

6. State and prove mean value theorem for Banach space.

7. Let X and Y be any two Banach spaces over the same field K. In the set of all functions tangential to a function f at $v \in V$, then show that there is most one function $\phi : X \to Y$, of the form $\phi(x) = f(v) + g(x - v)$, where $g : X \to Y$ is linear, where V is an non-empty open subset of X.