MAMT–05/MSCMT–05

December – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Mechanics)

Paper : V

Paper : MAMT–05/MSCMT-05

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Write formula for product of Inertia of rectangular plate of mass M and length of sides $2a$, $2b$ about its sides.

   (ii) Define Simple Equivalent Pendulum.

   (iii) Write vector form of Euler's equation of motion.

   (iv) State principle of conservation of angular momentum.

   (v) State Hamilton's principle.

   (vi) Define Viscosity.

   (vii) Define steady and unsteady flow.

   (viii) Define rotational and irrotational motion.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Show that the centre of suspension and centre of oscillation are convertible.

3. A uniform solid cylinder is placed with its axis horizontal on a place. Whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane, so that it may roll and not slide is $\frac{1}{3}\tan\alpha$.

4. Prove that the kinetic energy of rigid body, moving in any manner is at any instant equal to the kinetic energy of the whole mass, supposed to be collected at its centre of inertia and moving with it together with the kinetic energy of the whole mass relative to its centre of inertia.

5. Use Lagrange's equations to find the equation of motion of a simple pendulum.

6. Find the stream lines and path lines of the particles of the velocity field :

$$u = \frac{x}{(1+t)}, v = y$$

and $w = 0$.

7. A mass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinders, show that the equation of continuity is :

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho u)}{\partial \theta} + \frac{\partial(\rho v)}{\partial z} = 0$$

where u, v are the velocity perpendicular and parallel to z.

8. Air, obeying Boyle's law is in motion in a uniform tube of small section, prove that if ρ be the density and v be the velocity at a distance x from a fixed point at time t then :

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left\{(v^2 + k)\rho\right\}$$

9. In the case of the two-dimensional fluid motion produce by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O. Show that the velocity of slip

of the fluid in contact with the disc is greatest at the points where the line joining S to the ends diameter at right angles to OS cut the circle and prove that its magnitude at these points is :

$$\frac{2m.\mathrm{OS}}{(\mathrm{OS}^2 - a^2)}$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. A rod of length $2a$ is suspended by a string of length l, attached to one end; if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that :

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$$

11. A rectangular parallelopiped whose edges are a, $2a$, $3a$ can turn freely about its centre and is set rotating about a line perpendicular to the mean axis and making an angle $\cos^{-1}\frac{5}{8}$ with the least axis. Prove that ultimately the body will rotate about mean axis.

12. A symmetrical top is set in motion on a rough horizontal plane with an angular motion about its axis of figure, the axis being inclined at an angle i to the vertical. Show that between the greatest approach to and recess from the vertical, the centre of gravity describe on arc $h\tan^{-1}\left(\frac{\sin i}{p - \cos i}\right)$, where p and h have their usual meanings.

13. Show that $\phi = (x - t)\ (y - t)$ respresent the velocity potential of an incompressible two-dimensional fluid. Show that the stream line at time 't' are the curves $(x - t)^2 - (y - t)^2$ = constant and the path of the fluid particles have the equation :

$$\log(x - y) = \frac{1}{2}\left[(x + y) - a(x - y)^{-1}\right] + b$$

where a and b are constants.