Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. For the curve :

$$x = a(3u - u^3),\ y = 3au^2,\ z = a(3u + u^3)$$

show that the curvature and torsion are equal.

11. (a) Prove that on a given surface, a family of curves and their orthogonal trajectories can always be chosen as parametric curves.

(b) Show that the divergence of Einstein tensor vanishes.

12. Prove that an entity whose inner product with an arbitrary tensor is a tensor, is itself a tensor.

13. Show that the metric of a Euclidean space, referred to spherical coordinates is given by :

$$ds^2 = (dr)^2 + (rd\,\theta)^2 + (r \sin\theta\, d\,\phi)^2$$

Determine its metric tensor and conjugate metric tensor.

MAMT–04/MSCMT–04

December – Examination 2023

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Geometry and Tensor)

Paper : MAMT–04/MSCMT–04

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A 8×2=16

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in maximum up to **30** words. Each question carries 2 marks.

1. (i) Find the equation to the tangent at the point θ on the circular helix :

$$\vec{r} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j} + c\theta\,\hat{k}$$

(ii) Define Torsion.

(iii) Define Ruled Surface.

(iv) Show that the surface $z - c = \sqrt{xy}$ is developable.

(v) State Meuniur's theorem.

(vi) Define Normal angle.

(vii) Define tensor of zero order.

(viii) Show that the covariant differentiation of invariants is commutative.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find the inflexional tangents at (x, y, z) on the surface $y^2z = 4ax$.

3. Prove that the indicatrix at a point of the surface $z = f(x, y)$ is a rectangular hyperbola if :

$$(1 + p^2)t + (1 + q^2)\,r - 2pqs = 0$$

4. Prove that the generators of a developable surface are tangents to curve.

5. Show that the metric of a surface is invariant under parametric transformation.

6. Show that to a given direction there is one and only one conjugate direction. Also derive the condition for the two directions (du, dv) and (Du, Dv) to be conjugate.

7. State and prove Bonnet's theorem for parallel surfaces.

8. Prove that :

$$\text{div}A_i = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^y}\left\{\sqrt{g}\,g^{rk}A_k\right\} = \text{div } A^i$$

where A^i and A_i are the contravariant and covariant components of the same vector A.

9. Show that on the surface of a sphere, all great circles are geodesics while no other circle is a geodesic.