MAMT–10/MSCMT–10

December – Examination 2022

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Mathematical Programming)

Paper : MAMT–10/MSCMT–10

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) What is a Hyperplane ?

   (ii) What will the outcome of the intersection of two convex sets ?

   (iii) Write down the formula for initial basic feasible solution.

   (iv) Write the introducing surplus variables in the following LPP :

   Min. :

   $$Z = x_1 + 2x_2$$

   S.t. :

   $$2x_1 + 5x_2 \geq 6$$

   $$x_1 + x_2 \geq 2$$

   $$x_1, x_2 \geq 0$$

   (v) Define Lagrange's function.

   (vi) Define Saddle Point.

(vii) Write Kuhn-Tucker conditions for the following non-linear programming problem :

Max. :

$$f(x) = 8x - x^2$$

S.t. :

$$x \leq 3$$

$$x \geq 0$$

(viii) What do you mean by Bellman's principle of optimality ?

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Show that $f(x) = 2x_1^2 + x_2^2$ is a convex function over R^2.

3. Explain Gomory's Mixed L.P.P. method or Fractional Cut method.

4. Explain the difference between continuous and integer programming.

5. Solve the following quadratic programming problem using Wolfe's method :

Min. :

$$f(x_1, x_2) = 4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$$

S.t. :

$$2x_1 + x_2 \leq 6$$

$$x_1 - 4x_2 \leq 0$$

$$x_1,\ x_2 \geq 0$$

6. Prove that the set of all optimal solution (global maximum) of the general convex programming problem is a convex set.

7. Test the definiteness of the quadratic form :

$$X^T AX = (x_1, x_2, x_3)\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

8. Use Bellman's optimality principle to divide a positive quantity '*b*' into *n* parts in such a way that their product is maximum.

9. Solve by dynamic programming :

Max. : $Z = x_1 + 9x_2$

S.t. : $2x_1 + x_2 \leq 25$

$x_2 \leq 11$

and $x_1, x_2 \geq 0$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Use bounded variable technique to solve the following LPP :

Max. : $Z = 2x_1 + x_2$

S.t. : $x_1 + 2x_2 \leq 10$

$x_1 + x_3 \leq 6$

$x_1 - x_2 \leq 2$

$x_1 - 2x_2 \leq 1$

$0 \leq x_1 \leq 3$

$0 \leq x_2 \leq 2$

11. Solve the following integer programming problem using branch and bound technique :

Max. :

$$Z = x_1 + x_2$$

S.t. :

$$3x_1 + 2x_2 \leq 12$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

12. Solve the following quadratic programming problem using Beale's method :

Max. :

$$f(x_1, x_2) = 2x_1 + 3x_2 - 2x_1^2$$

S.t. :

$$x_1 + 4x_2 \leq 4$$

$$x_1 + 2x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

13. Find the optimal solution of the following convex separable programming problem :

Max. :

$$Z = 3x_1 + 2x_2$$

S.t. :

$$4x_1^2 + x_2^2 \leq 16$$

and $x_1, x_2 \geq 0$