MAMT–09/MSCMT–09

December – Examination 2022

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MAMT–09/MSCMT–09

Time : ***3*** *Hours*] [*Maximum Marks :* ***80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define function of class A.

(ii) Find inverse Laplace transform of

$$\frac{pe^{-ap}}{p^2 - w^2},\ a > 0 .$$

(iii) Write Dirichlet's conditions.

(iv) Define Fourier complex transform.

(v) State faltung theorem for the Mellin transform.

(vi) State Parseval's theorem for Hankel transform.

(vii) Define integral equation of convolution type.

(viii) State Hilbert-Schmidt theorem.

Section–B **4×8=32**

(Short Answer Type Questions)

***Note* :–** Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that :

$$L\left[\int_t^\infty \frac{e^{-u}}{u} du;\ p\right] = \frac{\log(p+1)}{p}$$

3. Find :

$$L^{-1}\left[\frac{p^2}{p^4+4a^4}\right]$$

4. Find the Fourier sine and cosine transform of :

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 2-t, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

5. Solve the integral equation :

$$g(x) = 1 + \int_0^x \sin(x-t)\, g(t)\, dt$$

and verify your answer.

6. Solve by the method of successive approximation :

$$g(x) = \frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2} + \frac{1}{2}\int_0^1 t\, g(t)\, dt$$

7. Using Hilbert-Schmidt theorem, find the solution of the symmetric integral equation :

$$g(x) = 1 + x^2 + \frac{3}{2}\int_{-1}^1 (xt + x^2t^2)\, g(t)\, dt$$

8. Find the Hankel transform of the function taking $xJ_n(px)$ as the Kernel :

$$f(x) = \begin{cases} x^n, & 0 < x < a \\ 0, & x > a \end{cases} \qquad n > -1$$

9. Using Fredholm theory, solve :

$$g(x) = \cos 2x + \int_0^{2\pi} \sin x \cos t . g(t) dt$$

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Solve :

$$ty'' + (t - 1)y' - y = 0,$$

$$y(0) = 5, \; y(\infty) = 0$$

11. Obtain the Mellin transform of :

$$f(x) = \frac{2(1-x^2)^{\lambda-1} H(1-x)}{\Gamma(\lambda)},$$

$$g(x) = \frac{2(1-a^2x^2)^{\mu-1} H(1-ax)}{\Gamma(\mu)}$$

with $\lambda > 0$, $\mu > 0$, $0 < a < 1$. Hence or otherwise establish that :

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma\left(\frac{z}{2}\right) \Gamma\left(\alpha - \frac{z}{2}\right) a^z}{\Gamma\left(\beta + \frac{z}{2}\right) \Gamma\left(\gamma - \frac{z}{2}\right)} dz = \frac{2a^{2a}}{\Gamma(\alpha+\beta)\Gamma(\gamma-\alpha)}$$

$$\times \; {}_2F_1(\alpha, 1 + \alpha - \gamma; \alpha + \beta; a^2)$$

with $0 < \alpha < 1$, $0 < \alpha < \gamma$, $\beta > 0$.

12. Find the potential $V(r, z)$ of a field due to a flat circular disc of unit radius with its centre at the origin and axis along the z-axis satisfying the differential equation :

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0, \quad 0 \le r \le \infty, \; z \ge 0$$

and satisfying the boundary conditions $V = V_0$, when $z = 0$, $0 \le r \le 1$ and $\frac{\partial V}{\partial z} = 0$, when $z = 0$, $r > 1$.

13. Find the eigenvalues and eigen function of the homogeneous integral equation :

$$g(x) = \lambda \int_0^{\pi} \left[\cos^2 x \cos 2t + \cos 3x \cos^3 t\right] g(t)\,dt$$