MAMT–05/MSCMT–05

December – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Mechanics)

Paper : MAMT–05/MSCMT-05

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) State theorem of parallel axes for moment of inertia.

   (ii) Define Simple pendulum.

   (iii) Write Euler's dynamical equations of motion.

   (iv) What do you mean by degree of freedom ?

   (v) Define Isotropic fluid.

   (vi) Define rotational flow and irrotational flow.

   (vii) Write equation of continuity in spherical polar coordinates.

   (viii) Define conservative field of force.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Show that the centre of suspension and centre of oscillation are convertible.

3. Prove that for a rigid body moving about a fixed point $\frac{dT}{dt} = \vec{W}.\vec{G}$, where $\vec{G}$ is the moment of external forces about fixed point and $T = \frac{1}{2}\vec{H}.\vec{W}$, where $\vec{H}$ is the angular momentum about the fixed point.

4. If the earth be regarded as a solid of revolution, whose principal moments of inertia at its centre of gravity are A, A, C. Show that its axis of rotation describes a cone of very small angle about the axis of the figure in period $\frac{A}{C-A}$ siderial days.

5. A uniform rod, of length $2a$, is placed with one end in contact with a smooth horizontal table and is then allowed to fall, if α be the initial inclination to the vertical, show that its angular velocity when it is inclined at angle θ is $\left\{\frac{6g}{a}.\frac{\cos\alpha - \cos\theta}{1+3\sin^2\theta}\right\}^{1/2}$. Find also the reaction of the table.

6. Explain how Lagrange's equations are used in case of small oscillations.

7. Deduce Lagrange's equations from Hamilton's principle.

8. Derive Euler's dynamical equations of motion in Cartesian coordinates.

9. In the case of the two-dimensional fluid motion produced by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O. Show that the velocity of slip of the fluid in contact with the disc is greatest at the points where the line joining S to the ends diameter at right angles to OS cut the circle and prove that its magnitude at these points is $\frac{2m.OS}{(OS^2 - a^2)}$.

Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizontal, and a man of mass M′, starting from the upper end, walks down the plank so that is does not move; show that he gets to the other end in time $\sqrt{\frac{2\mathrm{M}'a}{(\mathrm{M}+\mathrm{M}')g\sin\alpha}}$, where a is the length of the plank.

11. Two unequal smooth spheres are placed one on the top of the other in unstable equilibrium, the

lower sphere resting on a smooth table. If the system is slightly disturbed, then prove that the spheres will separate when the line joining their centres makes an angle θ with the vertical given by the equation :

$$\frac{m}{m+\mathrm{M}}\cos^3\theta - 3\cos\theta + 2 = 0$$

Where M is the mass of the lower sphere and m is mass of the upper sphere.

12. When the axis of a symmetrical top is stationary and then spin is large and equal to n, a blow J is applied perpendicular to the axis at a distance d from the fixed point. Prove that the maximum angular deflection of the axis is approximately $2\tan^{-1}\left(\frac{\mathrm{J}d}{\mathrm{C}n}\right)$, C being the moment of inertia of the top about its axis of symmetry.

13. Show that $\phi = (x - t)(y - t)$ represents the velocity potential of an incompressible two dimensional fluid. Show that the stream line at time 't' are the curves $(x - t)^2 - (y - t)^2 =$ constant and the path of the fluid particles have the equation $\log(x - y) = \frac{1}{2}\left[(x+y) - a(x-y)^{-1}\right] + b$, where a and b are constants.