Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Solve :

$$xz^3dx - zdy + 2ydx = 0$$

11. Find the characteristics of the following :

(i) $y^2r - x^2t = 0$

(ii) $x^2r + 2x\,sy + y^2t = 0$

12. Find the series solution of the Gauss Hypergeometric equation :

$$x(1-x)\frac{d^2y}{dx^2}+[\gamma-(1+\alpha+\beta)x]\frac{dy}{dx}-\alpha\beta\gamma=0$$

In the neighbourhood of the regular singular points :

(i) $x = 0$

(ii) $x = 1$

13. Prove that :

$$\int_0^\infty e^{-x}L_m(x)L_n(x)dx = \delta_{mn} = \begin{cases} 0, & \text{if} \quad m \neq n \\ 1, & \text{if} \quad m = n \end{cases}$$

MAMT–03/MSCMT–03

December – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variations and Special Function)

Paper : MAMT–03/MSCMT–03

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A 8×2=16

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Write down the general form of second order partial differential equation in two independent variables x and y.

(ii) Write the λ equation in Monge's Method for solving $2s + (rt + s^2) = 1$.

(iii) Write down the Legendre Equation.

(iv) Write Gauss's hypergeometric differential equation.

(v) Fill in the blanks for ${}_2F_1(a, b; c; 0) =$ ....

(vi) Define Linear Functional.

(vii) Write generating function for Hermite polynomial.

(viii) Write Bessel's function of first kind of index n.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Solve :

$$\sin^3 y \frac{d^2 y}{dx^2} = \cos y$$

3. Solve the partial differential equation $4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$, where $u = 3e^{-x} - 3^{5x}$ when $t = 0$ by method of separation of variables.

4. Find the shape of the curve on which a bead is sliding from rest and accelerated by gravity will ship (without friction) in least time from one point to another.

5. Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to Canonical form and solve it.

6. Find the eigenvalues and eigen functions for the boundary value problem :

$$y'' - 3y' + 2(1 + \lambda)y = 0,\ y(0) = 0,\ y(1) = 1$$

7. Find the extremals of the functional :

$$F[y(x)] = \int_0^1 \sqrt{1 + y'^2}\, dx,\ y(0) = 0,\ y(1) = 2$$

8. Prove :

$${}_2F_1[a, b; 1 - a + b; -1) = \frac{\Gamma(1 - a + b)\Gamma\left(1 + \frac{b}{2}\right)}{\Gamma(1 + b)\Gamma\left(1 + \frac{b}{2} - a\right)}$$

9. Show that :

$$(2n + 1)(1 - x^2)Q_n' = n(n + 1)(Q_{n-1} - Q_{n+1})$$