Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Let N be an arbitrary normed linear space. Then prove that for each vector x in N induced a functional F, on N** defined by $F_s(s) = f(x)$ $\forall$ $f \in N^*$ such that $\| F_s \| = \| x \|$.

11. If B and B′ be Banach spaces and T is a continuous linear transformation of B into B′, then prove that T is continuous if and only if its graph is closed.

12. State and prove Spectral theorem.

13. State and prove the implicit function theorem.

MAMT–06/MSCMT–06

December – Examination 2022

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MAMT–06/MSCMT–06

*Time : **3** Hours*] [*Maximum Marks :* ***80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define bounded linear transformation for normed vector space.

(ii) State open mapping theorem.

(iii) State Parseval's identity for a Hilbert space.

(iv) Define Eigen value and Eigen vector of an operator.

(v) Define Ortho-normal set.

(vi) State Lipschitz's function in a Banach space.

(vii) Define derivative of a map.

(viii) State Polarisation identity in a Hilbert space.

Section–B **4×8=32**

(Short Answer Type Questions)

***Note* :–** Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Prove that every normed linear space is a metric space.

3. If M be a closed linear sub-space of a normed linear space N and x_0 is a vector not in M, then prove that ∃ a functional F in conjugate space N* s.t. F(M) = {0} and $F(x_0) \neq 0$.

4. Prove that a closed convex subset K of a Hilbert space H contains unique vectors of smallest norm.

5. State and prove Bessel's inequality in Hilbert spaces.

6. State and prove mean value theorem for Banach space.

7. Let X and Y be any two Banach spaces over the same field K. In the set of all functions tangential to a function f at $v \in V$, then show that there is most one function $\phi : X \to Y$, of the form $\phi(x) = f(v) + g(x - v)$, where $g : X \to Y$ is linear, where V is an non-empty open subset of X.

8. If x and y are any two vectors in an inner product space X, then show that :

$$|(x, y)| \leq \| x \| \, \| y \|.$$

9. Let f be a regulated function on a compact interval $[a, b]$ of R into a Banach space X. Then prove that at each $t \in [a, b]$, the function $F : [a, b] \to X$, $F(t) = \int_a^t f,\ t \in [a, b]$ is continuous.