13. Show that it is always possible to choose a coordinate system so that all the Christoffel symbols vanish at a particular point.

MAMT–04/MSCMT–04

December – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Geometry and Tensors)

Paper : MAMT–04/MSCMT–04

*Time : **3** Hours*] [*Maximum Marks :* ***80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Bertrand curves.

(ii) Define Indicatrix.

(iii) Define principal section and principal direction.

(iv) State Beltrami-Enneper theorem.

(v) Define Geodesic.

(vi) Write Gauss' characteristic equations.

(vii) Define skew symmetric tensor.

(viii) Show that the covariant differentiation of invariants is commutative.

Section–B 4×8=32

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Find the inflexional tangents at (x, y, z) on the surface $y^2z = 4ax$.

3. Prove that each characteristic of a surface touches the edge of regression.

4. Show that the metric of a surface is invariant under parametric transformation.

5. Find the asymptotic lines on the surface $z = y \sin x$.

6. Show that an entity whose inner product with an arbitrary tensor is a tensor, is itself a tensor.

7. Prove that the fundamental tensor g_{ij} is a covariant symmetric tensor of the order two.

8. Show that the Christoffel symbols are not tensor quantities.

9. Show that the divergence of Einstein tensor vanishes.

Section–C 2×16=32

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. Show that the curvature and torsion are equal for the curve :

$$x = a(3u - u^3),\ y = 3au^2,\ z = a(3u + u^3)$$

11. Find the curvature of a normal section of the right helicoid $x = u \cos \phi$, $y = u \sin \phi$, $z = c\,\phi$.

12. If parameter s (arc length), then show that geodesic curvature $\kappa_g = [\hat{N}\ \vec{r}'\ \vec{r}'']$ and if we replace parameter s by t, then show that :

$$\kappa_g - \frac{1}{H\dot{s}^3}\left\{\frac{\partial T}{\partial \dot{u}} V(t) - \frac{\partial T}{\partial \dot{v}} U(t)\right\}$$