MAMT–02/MSCMT–02

December – Examination 2022

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Real Analysis and Topology)

Paper : MAMT-02/MSCMT-02

*Time : **3** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into three Sections A, B and C. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Measurable set.

 (ii) Define Directed set.

(iii) Define Orthonormal system.

(iv) State Weierstrass approximation theorem.

(v) State Holder's inequality.

(vi) What do you mean by complete orthonormal system ?

(vii) Define compact topological space.

(viii) Define Nets.

Section–B **4×8=32**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 8 marks.

2. Let E is a measurable set, then for any set A show that :

$$m^*(E \cup A) + m^*(E \cap A) \leq m^*(E) + m^*(A)$$

3. Let f be a measurable function finite on $E = [a, b]$. Then prove that for given $\in > 0$, there exists a function φ, continuous on $[a, b]$ such that :

$$m(\{x \in E : f(x) \neq \varphi(x)\}) < \in.$$

4. If a function is summable on E, then show that it is finite almost everywhere on E.

5. State and prove Minkowski's inequality.

6. Prove that homeomorphism is an equivalence relation in the family of topological spaces.

7. Show that regularity is a topological property.

8. Show that a function $f : X \to Y$ is continuous iff the inverse of each member of a base **B** for Y is an open subset of X.

9. Prove that every open continuous image of a locally compact space is locally compact.

Section–C **2×16=32**

(Long Answer Type Questions)

Note :– Answer any *two* questions. You have to delimit your each answer maximum up to **500** words. Each question carries 16 marks.

10. State and prove Weierstrass approximation theorem.

11. Prove that a subset of (R, U) is compact if and only if it is bounded and closed.

12. State and prove Riesz-Fischer theorem.

13. Show that a topological X is disconnected iff there exists a proper subset of X which is both open and closed in X.