MAMT-02/MSCMT–02

December – Examination 2021

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Real Analysis and Topology)

Paper : MAMT-02/MSCMT-02

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions.

1. (i) Define σ ring.

   (ii) Define Lebesgue measure of a set.

   (iii) Define orthonormal system.

(iv) State Riesz-Fisher theorem.

(v) Write the necessary and sufficient conditions for a bounded function f defined on the interval $[a, b]$, to be L-integrable.

(vi) State Parseval's identity.

(vii) Define base for a topology.

(viii) Define compact topological space.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each question carries 16 marks.

2. Let E is a measurable set, then for any set A show that :

$$m^*(E \cup A) + m^*(E \cap A) \le m^*(E) + m^*(A)$$

3. If the function f and g are Lebesgue integrable over the measurable set E and if $f(x) < g(x)$ on E, then prove that :

$$\int_E f(x)dx \le \int_E g(x)dx$$

4. Show that an orthonormal system $\{\phi_i\}$ is complete iff it is closed.

5. State and prove Minkowski's inequality.

6. Prove that for a subset A of a topological space (X, τ), $\bar{A} = A \cup A'$.

7. Show that every metric space is a T_2-space.

8. Show that the property of a space being a Haudorff space is a hereditary property.

9. Prove that a subset of real numbers R is connected if and only if it is an interval.