MA/MSCMT–10

December – Examination 2021

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Mathematical Programming)

Paper : MA/MSCMT-10

Time : 1½ Hours] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A 4×4=16

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) A hyperplane is given by the equation $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$. Find in which half spaces do the point (1, 2, 4, 1) lies.

(ii) Define positive semi-definite and negative semi-definite quadratic forms.

(iii) Write standard form –I for revised simplex method of linear programming problem :

Max. :

$$Z = CX$$

Subject to :

$$AX = b$$

$$X \geq 0$$

(iv) Explain importance of integer programming problem.

(v) Write steps of fractional cut method for solving integer programming problem.

(vi) Test the definiteness of the quadratic form :

$$X^T AX = (x_1, x_2, x_3)\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(vii) Define constrained optimization problem and unconstrained optimization problem.

(viii) Write dual of quadratic forms when X is unrestricted in sign.

Max. :

$$f(X)$$

Subject to :

$$g_i(X) = b_i, \; i = 1, 2, \ldots\ldots\ldots, m$$

Section–B 4×16=64

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Solve the following linear programming problem using revised simplex method till first revised simplex table :

Max. :

$$Z = 3x_1 + 6x_2 + 2x_3$$

Subject to :

$$3x_1 + 4x_2 + x_3 \leq 2$$

$$x_1 + 3x_2 + 2x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

3. Use Branch and Bound Method to solve the following integer programming problem where x_1 and x_2 are integers :

Max. :

$$Z = 4x_1 + 3x_2$$

Subject to :

$$5x_1 + 3x_2 \geq 30$$

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

4. Find the dimension of a rectangular parallelopiped with largest volume whose sides are parallel to the coordinate planets, to be inscribed in the ellipsoid.

5. Use Kuhn-Tucker conditions to solve the following non-linear programming problem :

   Max. :

$$f(x_1, x_2) = 7x_1^2 - 6x_1 + 5x_2^2$$

   Subject to :

$$x_1 + 2x_2 \leq 10$$

$$x_1 - 3x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

6. Solve the following quadratic programming problem by Beale's method :

   Min. :

$$f(x_1, x_2) = 10x_1^2 + x_2^2 + 4x_1x_2 - 10x_1 - 25x_2$$

   Subject to :

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

7. Prove that every local maximum of the general convex programming problem is its global maximum.

8. Use dynamics programming to solve :

   Min. :

$$\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)$$

   Subject to :

$$x_1, x_2, \ldots, x_n = b$$

   and $x_1, x_2, \ldots, x_n \geq 0$

9. Solving the following linear programming problem by using dynamic programming :

Max. :

$$Z = 3x_1 + 5x_2$$

Subject to :

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 18$$

and $$x_1, x_2 \geq 0$$