7. Solve $\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$, $x > 0$, $t > 0$, if :

   (i) $V_x(0, t) = 0$

   (ii) $V(x,0) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

   (iii) $V(x, t)$ is bounded

8. Solve the integral equation :

$$g(x) = x + \lambda \int_{-\pi}^{\pi} \left(x \cos t + t^2 \sin x + \cos x \sin t \right) g(t) dt$$

9. Find the resolvent kernel of the volterra integral equation with the kernel :

$$K(x,t) = \frac{(2 + \cos x)}{(2 + \cos t)}$$

MA/MSCMT–09

December – Examination 2021

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Integral Transforms and Integral Equations)

Paper : MA/MSCMT-09

*Time : **1½** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the questions delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Prove that :

$$L\left\{\left((t-3)^2+6(t-3)+9\right).u(t-3)\right\}$$

$$= e^{-3p}\left(\frac{2+6p+9p^2}{p^3}\right)$$

(ii) Find :

$$L^{-1}\left\{\frac{(p+3)}{\left(p^2+6p+45\right)}\right\}$$

(iii) State Parseval's identity for Fourier transform.

(iv) If $M[f(x); p] = F(p)$, then prove that :

$$M\left[f\left(x^a\right); p\right] = \frac{1}{a}F\left(\frac{p}{a}\right)$$

(v) Define Hankel transform and its kernel.

(vi) Define Volttera integral equation of third kind.

(vii) Define degenerated kernel.

(viii) Define complex Hilbert space.

Section–B 4×16=64

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. If $f(t)$ is a periodic function with period $T > 0$, then prove that :

$$L[f(t); p] = \frac{\int_0^T e^{-pt} f(t)dt}{1-e^{-pT}}$$

3. Use complex inversion formula to obtain the inverse Laplace transform of :

$$\frac{p}{(p+1)(p-1)^2}$$

4. Using Laplace transform solve $(2D^2 + 3D - 2)y = 0$ given $y(0) = 1$, $y(t) \to 0$ as $t \to \infty$.

5. Find $f(t)$ if its Fourier sine transform is $\frac{p}{\left(1+p^2\right)}$.

6. Find the Hankel transform of :

(a) $\frac{\cos ax}{x}$

(b) $\frac{\sin ax}{x}$

Taking $xJ_0(px)$ as kernel.