8. If $[a, b]$ be a compact interval and g be a regulated function on $[a, b]$ into $\{r \in R;\ r \geq 0\}$ and if h be a continuous function on $[a, b]$ into R for all $t \in [a, b]$:

$$h(t) \leq g(t) + c\int_a^t h(s)ds$$

Where c is a positive real number, then show that :

$$\forall t \in [a,b],\ h(t) \leq g(t) + c\int_a^t g(s)e^{c(t-s)}ds$$

9. State and prove existence theorem for solution of differential equations on Banach space.

MA/MSCMT–06

December – Examination 2021

M.A./M.Sc. (Final) Examination

MATHEMATICS

Paper : MA/MSCMT-06

Time : 1½ Hours] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the questions delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Prove that limit of a convergent sequence is unique.

   (ii) State Holder's inequality.

   (iii) State open mapping theorem.

   (iv) Define closed linear transformation.

   (v) Define first dual space.

   (vi) State Hanh-Banach theorem.

   (vii) Define directional derivatives.

   (viii) Define derivatives of a map.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. If N be a normed linear space, then show that the two norms $\| \|_1$ and $\| \|_2$ defined on N are equivalent if and only if there exists positive real numbers a and b such that $a\|x\|_1 \le \|x\|_2 \le b\|x\|_1$, $\forall x \in N$.

3. If x and y are any two vectors in an inner product space X, then prove that $|(x, y)| \le \|x\|\|y\|$.

4. Prove that every Hilbert space is reflexive.

5. Prove that an oeprator T on a Hilbert space H is unitary if and only if it is an isometric isomorphism of H onto itself.

6. If P and Q are projections on closed linear subspaces M and N of a Hilbert space H, then show that M ⊥ N ⇔ PQ = 0 ⇔ QP = 0.

7. If X be a Banach space over the field K of scalars, and I be an open interval in R containing [0, 1]. If ψ : I → X is (n + 1) times continuously differentiable function of a single variable $t \in$ I then prove that :

$$\psi(1) = \psi(0) + \psi'(0) + \frac{\psi''(0)}{2!} + \ldots\ldots + \frac{\psi^n(0)}{n!} + \int_0^1 \frac{(1-t)^n}{n!}\psi^{n+1}(t)dt$$