8. Prove that :

$$A^{ij}_{,j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j}\left(\sqrt{g} A^{ij}\right) + A^{jk} \begin{Bmatrix} i \\ jk \end{Bmatrix}$$

Show that the last term vanishes if $A^{ij}$ is skew-symmetric.

9. Obtain the differential equations of geodesics for the metric :

$$ds^2 = f(x) dx^2 + dy^2 + dz^2 + \frac{1}{f(x)} dt^2$$



# MA/MSCMT–04

## December – Examination 2021

# M.A./M.Sc. (Previous) Examination

## MATHEMATICS

## (Differential Geometry and Tensor)

## Paper : MA/MSCMT-04

*Time : 1½ Hours* ] [ *Maximum Marks : **80***

***Note*** :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

**Section–A** **4×4=16**

**(Very Short Answer Type Questions)**

***Note*** :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Define Torsion.
   (ii) Define Conoids.
   (iii) Define Anchor ring.
   (iv) Write geometrical significance of second fundamental form.
   (v) Examine whether the parametric curves $x = b \sin u \cos v$, $y = b \sin u \sin v$, $z = b \cos u$ on a sphere of radius $b$ constitute an orthogonal system.
   (vi) Define geodesic curve.
   (vii) Define summation convention.
   (viii) Define absolute derivative.

**Section–B** **4×16=64**

**(Short Answer Type Questions)**

***Note*** :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Show that when the curve is analytic, there exists a definite osculating plane at a point of inflexion, provided the curve is not a straight line.



3. Prove that a curve is uniquely determined, except as to position in space, when its curvature and torsion are given functions of its arc-length.

4. Prove that every characteristic curve of family of surfaces touches the edge of regression.

5. Prove that there are *two* principal directions at every point on a surface which are mutually orthogonal.

6. Show that conjugate directions at a point P on a surface are parallel to conjugate diameters of the indicatrix at P.

7. Show that the metric of a Euclidean space, referred to cylindrical coordinates is given by $ds^2 = (dr)^2 + (rd\theta)^2 + (dz)^2$. Determine its metric tensor and conjugate metric tensor.