7. Discuss the relation between $J_n(x)$ and $J_{-n}(x)$, n being an integer.

8. Show that :

$$\left(1-2xh+h^2\right)^{-\frac{1}{2}} = \Sigma_{n=0}^{\infty} P_n(x)h^n \quad |x| \le 1,\ |h| \le 1$$

9. Expand x^n in a series of Hermite Polynomials.

MA/MSCMT–03

December – Examination 2021

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Differential Equations, Calculus of Variations and Special Functions)

Paper : MA/MSCMT-03

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A 4×4=16

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Write condition of integrability for a total differential equation.

(ii) Define an isoperimetric problem.

(iii) Classify the following PDE as hyperbolic, parabolic or elliptic :

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$

(iv) Write generating function for Bessel function.

(v) Write Monge's subsidiary equations for partial differential equation $pt - qs = q^3$.

(vi) Write the condition for which the following Partial Differential Equation is elliptic :

$$Rs + Ss + Tt + F(x, y, z, p, q) = 0$$

(vii) Write Gauss's Hypergeometric differential equation.

(viii) Write orthogonal property for Legendre polynomial.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. Solve the following differential equation :

$$2\sin x \frac{d^2 y}{dx^2} + 2\cos x \frac{dy}{dx} + 2\sin x \frac{dy}{dx} + 2y\cos x = \cos x$$

3. Find the eigenvalues and eigenfunctions for the boundary value problem :

$$y'' - 2y' + \lambda y = 0;\ y(0) = 0,\ y(\pi) = 0$$

4. Find the characteristics of $x^2 r + 2xys + y^2 t = 0$.

5. Define Gauss's Hypergeometric Series and discuss its convergence conditions.

6. Show that :

$$\int_0^t x^{\frac{1}{2}} (t-x)^{-\frac{1}{2}} [1 - x^2 (t-x)^2]^{-\frac{1}{2}} dx$$

$$= \frac{1}{2} \pi t {}_2F_1 \left[\frac{1}{4}, \frac{3}{4};\ 1;\ \frac{t^4}{16} \right]$$