7. If $t : V \rightarrow V$ be a linear transformation from a finite dimensional vector space V to itself. Assume that v_i ; $i = 1, 2, ..., n$ are distinct eigen vectors of t corresponding to distinct eigen values λ_i ; $i = 1, 2, ..., n$. Then show that set $\{v_1, v_2, ..., v_n\}$ is a linearly independent set.

8. State and prove Schwartz inequality.

9. Show that every finite dimensional vector space V with an inner product has an orthonormal basis.

MA/MSCMT–01

December – Examination 2021

M.A./M.Sc. (Previous) Examination

MATHEMATICS

(Advanced Algebra)

Paper : MA/MSCMT-01

Time : 1½ Hours] [*Maximum Marks :* ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable Scientific Calculator is allowed in this paper.

Section–A **4×4=16**

(Very Short Answer Type Questions)

Note :– Answer any *four* questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 4 marks.

1. (i) Define Internal direct product of groups.

(ii) Define subnormal series.

(iii) Define quotient module.

(iv) Define nullity of linear map.

(v) Define algebraic extension of a field.

(vi) Define eigen values of a matrix.

(vii) Define inner product space.

(viii) Define self-adjoint linear map.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Each answer should not exceed **200** words. Each question carries 16 marks.

2. If G is a finite group, then show that the number of elements conjugate to $\underline{a}$ in G is the index of the normalizer of a in G, i.e. $|C[a]| = |G : N(a)|$.

3. If G be a group with subgrops H and K such that :

(i) H and K are normal in G and

(ii) $H \cap K = \{e\}$

then show that HK is the informal direct product of H and K and $HK \cong H \times K$.

4. Prove that every Euclidean ring R is a principal ideal domain.

5. If F be a field and $p(x) \in F[x]$ such that $\deg p(x) = n$, where $n \geq 1$. Then show that there exists a finite extension K of F in which $p(x)$ gets a full set of n roots such that $[K : F] \leq n!$

6. If V and V′ be finite dimensional vector spaces over a field F and $t : V \to V'$ be a linear transformation, t^* is dual of t. Then show that the maps t and t^* have same rank.