7. Obtain the necessary and sufficient conditions for the optimum solution of the following non-linear programming problem :

Minimize :

$$Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

Subject to :

$$x_1 + x_2 + x_3 = 15$$
$$2x_1 - x_2 + 2x_3 = 20$$
$$x_1, x_2, x_3 \geq 0$$

8. Use Kuhn-Tucker conditions to determine x_1, x_2, x_3 so as to minimize $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 6x_2$

Subject to :

$$x_1 + x_2 \leq 2$$
$$2x_1 + 3x_2 \geq 12$$
$$x_1, x_2 \geq 0$$

9. (i) Determine which of the following equations are quadratic form :

(a) $Z = x_1^2 + 2x_2$ (b) $Z = x_1^2 - x_2^2$

(ii) Determine whether or not the quadratic forms X^TAX are positive definite, where :

(a) $A = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

MA/MSCMT–10

December – Examination 2020

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Mathematical Programming)

Paper : MA/MSCMT-10

*Time : **2** Hours*] [*Maximum Marks : **80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions. Use of non-programmable scientific calculator/simple calculator allowed in this paper.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define Convex Function.

(ii) Define Integer Programming Problem.

(iii) Write the objective function in the form :

$Z = X^T AX + q^T X$

where

$$Z = x_1^2 + 2x_1x_2 + 4x_1x_3 + 3x_2^2 + 2x_2x_3 + 5x_3^2 + 4x_1 - 2x_2 + 3x_3$$

(iv) How to determine that the quadratic objective function $f(x)$ is concave or convex by using the quadratic form $X^T GX$

(v) Define optimal solution of a mathematical programming problem.

(vi) Define Separable Function.

(vii) Prove that a hyperplane is a convex set.

(viii) Define Lagrange's Function.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Answer should not exceed **200** words. Each question carries 16 marks.

2. Prove that the optimal hyperplane in a linear programming problem is a supporting hyperplane to the convex set of feasible solutions.

3. Solve the following integer programming problem by branch and bound method :

Maximize : $Z = x_1 + x_2$

Subject to :

$$3x_1 + 2x_2 \le 12$$
$$x_2 \le 2$$
$$x_1, x_2 \ge 0 \text{ and integers.}$$

4. Solve the following programming problem graphically :

Minimize :

$$f(x_1, x_2) = x_1^2 + x_2^2$$

Subject to :

$$x_1 + x_2 \ge 4$$
$$2x_1 + x_2 \ge 5$$
$$x_1, x_2 \ge 0$$

5. Solve the following quadratic programming problem by Beale's mehtod :

Maximize :

$$f(x_1, x_2) = x_1 + x_2 - x_1^2 + x_1x_2 - 2x_2^2$$

Subject to :

$$2x_1 + x_2 \le 1$$
$$x_1, x_2 \ge 0$$

6. Use of dynamic programming to show that :

$$\sum_{i=1}^{n} p_i \log p_i$$

Subject to $\sum_{i=1}^{n} p_i = 1, p_i > 0$ is minimum when $p_1 = p_2 = \ldots\ldots\ldots\ldots = p_n = \frac{1}{n}$.