MA/MSCMT–06

December – Examination 2020

M.A./M.Sc. (Final) Examination

MATHEMATICS

(Analysis and Advanced Calculus)

Paper : MA/MSCMT-06

Time : **2** *Hours*] [*Maximum Marks* : ***80***

Note :– The question paper is divided into two Sections A and B. Write answers as per the given instructions.

Section–A **8×2=16**

(Very Short Answer Type Questions)

Note :– Answer all questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to **30** words. Each question carries 2 marks.

1. (i) Define weak convergence of a sequence.

(ii) State open mapping theorem.

(iii) Define natural embedding.

(iv) Define eigen value and eigen vector of an operator.

(v) Define Ortho-normal set.

(vi) Define inner product space.

(vii) Define derivative of a map.

(viii) State polarisation identity in a Hilbert Space.

Section–B **4×16=64**

(Short Answer Type Questions)

Note :– Answer any *four* questions. Answer should not exceed **200** words. Each question carries 16 marks.

2. State and prove Minkowaski's inequality.

3. Let M be a closed linear subspace of a Hilbert space H. Let x be a vector not in M and $d = d(x, M)$. Then prove that there exist a unique vector y_0 in M s.t. $|| x - y_0 || = d$.

4. Prove that a closed convex subset K of a Hilbert space H contains a unique vector of smallest norm.

5. Show that the set of unitary operators on a Hilbert space H, forms a multiplicative group.

6. State and prove mean value theorem for Banach space.

7. State and prove Global uniqueness theorem.

8. If x and y are any two vectors in an inner product space X, then show that :

$$|(x, y)| \leq || x || \, ||y||$$

9. Show that every compact subset of a normed linear space is bounded but its converse need not be true.