MA/MSCMT-10

December - Examination 2019

M.A./M.Sc. (Final) Mathematics Examination

Mathematical Programming

Paper - MA/MSCMT-10

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Use of non-programmable scientific calculator / simple calculator allowed in this paper.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Type Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum upto 30 words. Each question carries 2 marks.

1. i. Define hyperplane.

   ii. What is the importance of integer programming problem?

   iii. Define positive definate matric.

   iv. What do you mean by relative minimum?

   v. Write difference between convex programming problem and non-linear programming problem.

   vi. Define convex separable programming problem.

   vii. State Bellman's principle of optimality

   viii. Define quadratic programming problem.

Section - B $\quad 4 \times 8 = 32$

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2. Prove the $f(x)=\frac{1}{x}$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

3. Solve the following integer programming problem by branch and bound algorithm.

$$\min z = 2x_1 + 6x_2$$
$$\text{s.t.} \quad 3x_1 + x_2 \leq 5$$
$$4x_1 + 4x_2 \leq 9$$
$$x_1, x_2 \geq 0 \text{ and are integers.}$$

4. Find the dimensions of a rectangular parallelopiped with largest volume to be inscribed in ellipsoid.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}= 1$$

5. Solve the following non-linear programming problem graphically.

$$\max f(x_1,x_2) = x_1 + 2x_2$$
$$\text{s.t.} \quad x_1^2 + x_2^2 \leq 1$$
$$2x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

6. Derive the dual of quadratic programming problem.

$$\min f(X) = C^T X + \frac{1}{2} X^T G X$$
$$\text{s.t.} \quad AX \geq b$$

Where A is an m x n real matrix and G is an n x n real positive semidefinite asymmetric matrix.

7. Divide a quantity ‘b’ into n parts in such a way that their product is maximum.

8. Solve following linear programming problem by dynamic programming.

$$\max z = 3x_1 + 7x_2$$

$$\text{s.t.} \quad x_1 + 4x_2 \leq 8$$

$$x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

9. use Kuhn - Tucker conditions to solve the following non - linear programming problem.

$$\max f(x) = 8x - x^3$$

$$\text{s.t.} \quad x \leq 3$$

$$x \geq 0$$

Section - C $\qquad 2 \times 16 = 32$

(Long Answer Type Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10. Solve the following linear programming problem by revised simplex method.

$$\max z = 2x_1 + x_2$$

$$3x_1 + 4x_2 \leq 6$$

$$6x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

11. Solve the following integer programming problem.

$$\max z = 2x_1 + 10x_2 - 10x_3$$

$$\text{s.t.} \quad 2x_1 + 20x_2 + 4x_3 \le 15$$

$$6x_1 + 20x_2 + 4x_3 = 20$$

$x_1, x_2, x_3 \ge 0$ and are integers.

12. Solve the following quadratic programming problem by Beak's method.

$$\max f(x_1, x_2) = 2x_1 + 3x_2 - 2x_1^2$$

$$\text{s.t.} \quad x_1 + 4x_2 \le 4$$

$$x_1 + 2x_2 \le 2$$

$$x_1, x_2 \ge 0$$

13. Solve the following convex separable programming problem.

$$\min z = x_1^2 - 2x_1 - x_2$$

$$\text{s.t.} \quad 2x_1^2 + 3x_2^2 \le 6$$

and $x_1, x_2 \ge 0$

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