MA/MSCMT-09

December - Examination 2019

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **8 × 2 = 16**

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) If $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$ then find $L\left\{\frac{\sin 2t}{t}\right\}$

(ii) Find Inverse Laplace transform of $\frac{1}{(4s+3)}$

(iii) Define Fourier cosine transformation.

(iv) If $M\{f(x)\,;p\} = F(p)$ then find $M\{x^a\, f(x);\, p\}$

(v) If $H_v\{f(x)\,;p\} = F_v(p)$ then find $H_v\{f(ax);\, p\}$

(vi) Define separable Kernal.

(vii) Define Abel Integral equation.

(viii)Define resolvent Kernal.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) Find Laplace transform of $\left(\frac{1-\cos t}{t^2}\right)$

3) Prove that $L^{-1}\left\{\frac{e^{-1/p}}{\sqrt{p}};\ t\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$

4) Solve $(D^2+9)y = \cos 2t$ If $y(0)=1,\ y\left(\frac{\pi}{2}\right) = -1$

5) Prove that if n is a positive integer then

$$M\left[\left(x\frac{d}{dx}\right)^n f(x);\ p\right] = (-1)^n\ p^n\ F(p) \text{ where } M\left[f(x);\ p\right] = F(p)$$

6) Find Hankel transform of $x^v e^{-ax}$ taking $xJ_v(px)$ as the Kernel.

7) Transform $\frac{d^2y}{dx^2} + xy = 1 : y(0)=0,\ y(1)=1$ into an integral equation.

8) Prove that the characteristics numbers of a symmetric Kennel are real.

9) Find the resolvent Kernel of following Kernel

$$K(x, t) = (1 + x)(1 - t) : a = -1, b = 0$$

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum up to 500 words. Each question carries 16 marks.

10) Use Parseval's Identity to prove that

(a) $\displaystyle\int_0^\infty \frac{t^2\,dt}{(1+t^2)(4+t^2)} = \frac{\pi}{6}$

(b) $\displaystyle\int_0^\infty \frac{dt}{(1+t^2)^2} = \frac{\pi}{4}$

11) Find the potential $V(r, z)$ of a field due to a flat circular disc of unit radius with its centre at the origin and axis along the Z-axis satisfying the differential equation.

$\dfrac{\partial^2 v}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial v}{\partial r} + \dfrac{\partial^2 v}{\partial z^2} = 0,\ 0 \le r \le \infty, z \ge 0$ and satisfying the boundary conditions $V = V_0$ when $z = 0, 0 \le r < 1$ and $\dfrac{\partial v}{\partial z} = 0$ when $z = 0, r > 1$

12) (i) Solve the Integral equation

$$g(x) = x + \lambda \int_{-\pi}^{\pi} (x \cos t + t^2 \sin x + \cos x \sin t)\, g(t)\, dt$$

(ii) Solve the integral equation

$$g(x) = 1 + \int_{0}^{x} \sin(x - t)\, g(t)\, dt \text{ and verify your answer}$$

13) (i) Solve by iterative method

$$g(x) = 1 + \int_{0}^{\pi} \sin(x + t)\, g(t)\, dt$$

(ii) Solve the following integral equation

$$g(x) = x + \lambda \int_{0}^{1} (4xt - x^2)\, g(t)\, dt$$
