MA/MSCMT-06

December - Examination 2019

M.A./ M.Sc. (Final) Mathematics Examination

Analysis and Advanced Calculus

Paper - MA/MSCMT-06

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum upto 30 words. Each question carries 2 marks.

1. i. Explain convergence in normed linear space.

ii. Define conjugate space.

iii. Define natural embedding

iv. State spectral theorem.

v. State Risez representation theorem.

vi. Define inner product space.

vii. Define integral solution of the differential equation.

viii. Define regulated function.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2. Prove that every normed linear space is a metric space.

3. If M be a closed linear subspace of a normed linear space N and x_0 is a vector not In M, then prove that $\exists$ a functional F in conjugate space N^* s.t. $F(M) = \{0\}$ and $F(x_0) \neq 0$.

4. Prove that a closed convex subset K of a Hilbert Space H contains a unique vectors of smallest norm.

5. Prove that if S is a non empty subset of a Hilbert space H, then $S^\perp$ is a closed linear subspace of H.

6. If T is an operator on a Hilbert space H, then prove that $(Tx, x) = 0$ $(Tx, x) = 0$ if and only if $T = 0$.

7. If T is a normal operator on a Hibert space H then prove that
   i. each eigenspace of T reduces T and
   ii. eigenspaces of T are pairwise orthogonal.

8. State and prove inverse function theorem.

9. Let I be a closed interval in R, W be a closed set in a Banach space X and $g : I \times W \to X$ be a continuous function which is c-Lipschitz in $x \in X$ Let $(s, x_0) \in I \times W$ for given $r > 0$,

   Let $f : I \to X$ be an r-approximate solution of the differential equation $\frac{dx}{dt} = g\,(t, x)$ such that $f(s) = x_0$ then prove that there exists in I an exact solution of $\phi : I \to X$ the differential equation such that $\phi\,(s) = x_0$

Section - C $\quad 2 \times 16 = 32$

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10. If N and N' be normed linear spaces and $B(N, N')$ is the set of all bounded linear transformation from N into N' then prove that $B(N, N')$ is a normed linear space under the norm $||T||= \sup \{|T(x)|: ||x|| \leq 1\} \forall x \in N$ with respect to pointwise linear operations $(T+S)(x) = T(x) + S(x)$ and $(\alpha T)(x) = \alpha T(x)$, for real α. Also prove that $B(N, N')$ is complete if N' is complete.

11. If B and B' be Banach spaces and T a continuous linear transformation of B on to B', then prove that the image of every open sphere centred at origin in B contains an open sphere centred at origin in B'.

12. State mean value theorem for a mapping defined on a Banach space. If X be a Banach space over the field K of scalars and let $f : [a,b] \to X$ and $g : [a, b] \to R$ be continuous and differentiable functions such that $||Df(t)|| \leq Dg(t)$ at each point $t \in (a, b)$ then prove that $||f(b) - f(a)|| \leq g(b) - g(a)$

13. Let X and Y be Banach spaces over the same field K of scalars and V be an open subset of X. Let $f : V \to Y$ is twice differentiable at a point $v \in V$ then prove that $D^2 f(v) \in L(X^2, Y)$ is a bilinear symmetric mapping i.e. for all $(x, y) \in X \times X$, $D^2 f(v).(x, y) = D^2 f(v).(y, x)$