# MA/MSCMT-05

## December - Examination 2019

## M.A. / MSc. (Previous) Mathematics

## Examination

## Mechanics

## Paper - MA/MSCMT-05

**Time : 3 Hours ]** **[ Max. Marks :- 80**

**Note:** The question paper is divided into three sections A, B and C. Write answers as per given instructions.

### Section - A $8 \times 2 = 16$

### (Very Short Answer Questions)

**Note:** Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum upto 30 words. Each question carries 2 marks.

1. i. Find moment of inertia of hollow sphere of radius r about diameter.

   ii. Write vector form of Euler's equation.

   iii. State principle of conservation of linear momentum.

   iv. What do you mean by holonomus system?

   v. What is the difference between Hamilton's principle and principle of least action?

vi. Define stream filament.

vii. Define boundary surface.

viii. What do you mean by strength of source?

## Section - B $4 \times 8 = 32$

### (Short Answer Questions)

**Note:** Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2. State and prove D'Alem bert's principle.

3. Deduce Euler's equations from Lagrange's equations.

4. Show that for a body of revolution the maximum value of angle between axis of the impulsive couple acting on it and the instantaneous axis of initial motion set up by couple in the body is $\text{Sin}^{-1}\left(\frac{C-A}{C+A}\right)$

5. Prove that when a body moves under the action of a system of conservative forces, the sum of its Kinetic and potential energies is constant throughout the motion.

6. Find the stream lines and path lines of the particles of the velocity field.

   $U = \frac{x}{(1+t)}$, $V$ = y and w = 0

7. If every particle of a fluid moves on the surface of a sphere prove that the equation of continuity is

   $$\frac{\partial \ell}{\partial t}\cos\theta + \frac{\partial}{\partial \theta}(\ell\, w \cos\theta) + \frac{\partial}{\partial \phi}(\ell\, w^1 \cos\theta) = 0$$

   $\ell$ being density, $\theta$, $\phi$ the latitude and longitude of any element and w, $w^1$ the angular velocities of the element in latitude and longitude respectively.

8. Derive equation of continuity in vector form by Euler's method.

9. Explain image of a doublet with respect to a circle.

## Section - C $2 \times 16 = 32$

## (Long Answer Questions)

**Note:** Answer **any two** questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10. Two uniform equal rods AB and AC are freely hinged at A and rest in a straight line on a smooth table. A blow is struck at it perpendicular to the rods. show that kinetic energy generated is $\frac{7}{4}$ times what it would be if the rods were rigidly fastened together at A

11. Establish Lagrange's equations for impulsive forces.

12. An infinite mass of homogenous imcompressible fluid is at rest subject to a uniform pressure P and contains a spherical cavity of radius '$a$' filled with gas at a pressure MP. Prove that if the inertia of gas be neglected any Boyle's law be supposed to hold through out the ensuing motion, the radius of sphere will oscillate between the value a and na, where n is determined by the equation $1+3m \log n - n^3 = 0$

    If m is nearly equal to 1 the prove that the time of an oscillation will be $2\Pi, \sqrt{\frac{a^2 \ell}{3p}}$ being the density of fluid.

13. A sphere of radius a is surrounded by infinite liquid of density $\ell$, the pressure at infinity being $\Pi$. the sphere is suddenly annihilated. Show that the pressure at a distance $\gamma$ from the centre immediately falls to $\Pi\left(1-\frac{a}{\gamma}\right)$ show further that if the liquid is brought to rest by impinging on a concentric sphere of radius $\frac{a}{2}$, the impulsive pressure sustained by the surface of sphere is $\left(7\Pi\ell^2/6\right)^{1/2}$

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