MA/MSCMT-04

December - Examination 2019

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Geometry and Tensors

Paper - MA/MSCMT-04

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum upto 30 words. Each question carries 2 marks.

1. i. Define inflexional tangent.

   ii. Write equation of rectifying plane.

   iii. Show that the distance between corresponding points of two involutes is constant.

   iv. Define metric of a surface.

   v. Define Trajectory of given family of curve.

   vi. Define umbilic.

vii. Define symmetric tensor.

viii. State fundamental theorem of Riemannian geometry.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2. Find the inflexional tangent at point (x_1, y_1, z_1) of the surface $y^2z = 4ax$.

3. Find the developable surface which passes through the curve $y^2 = 4ax$, $z = 0$ and $y^2 = 4bz$, $x = 0$.

4. Prove that the metric of a surface is invariant under parametric transformation.

5. State and prove Meunier's theorem.

6. Prove that a curve on sphere is geodesic if and only if it is a great circle.

7. Prove that an entity whose inner product with an arbitrary tensor is a tensor, is itself a tensor.

8. Prove that the necessary and sufficient conditions that a system of co-ordinates be geodesic with the pole p_0 are that their second covariant derivatives, with respect to the metric of the space, all vanish at p_0.

9. Prove that if a Riemannian space V_N (N>2) is isotropic at each point in a region then the Riemannian curvature is constant throughout that region.

Section - C $2 \times 16 = 32$

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10. For the curve

    $x = a \cos\theta$, $y = a \sin\theta$, $z = a\,\theta \tan \propto$.

    Find :

    a. Radii of curvature 5

    b. Torsion of curve 5

    c. Involute of curve 6

11. a. On the paraboloid $x^2 - y^2 = z$, Find the orthogonal trajectories of the sections by the plane z = constant.

    b. Show that conjugate direction a a point P on a surface are parallel to conjugate diameters of the indicatrix at P.

12. a. Show that the Christoffel symbols are not tensor quantities.

    b. State and prove Bonnet's theorem.

13. Find the principal section and principal curvature of the surface $x = a(u+v)$, $y = b(u-v)$, $z = uv$.