MA/ MSCMT-03

December - Examination 2019

M.A. / M.Sc. (Previous) Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA/ MSCMT-03

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions. Use of non-programmable scientific calculator is allowed in this paper.

Section - A **$08 \times 02 = 16$**

(Very Short Answer Type Questions)

Note: Answer all Questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 02 mark.

1) (i) Write down Rodrogue's formula for the Laguerre polynomial.

(ii) Define isoperimetric problem.

(iii) Solve $y^3 \frac{d^2 y}{dx^2} = c$

(iv) Write two dimensional Laplace equation in polar coordinate system.

(v) Write Generating function for Hermite Polynomial.

(vi) Give a common method for solving Laplace, wave and diffusion equations.

(vii) Write Bessal's Function of First kind of index n.

(viii)Write the laguerre differential equation of order n.

Section - B **04 × 08 = 32**

(Short Answer Type Questions)

Note: Answer any four question. Each answer should not exceed 200 words. Each question carries 08 marks.

2) Show that the differential equation

$y + 3x\frac{dy}{dx} + 2y\left(\frac{dy}{dx}\right)^3 + \left(x^2 + 2y^2 \frac{dy}{dx}\right)\frac{d^2 y}{dx^2} = 0$ is an exact

equation, hence find its first integral.

3) Find the differential equation of family of twisted cubic curves $y = ax^2, y^2 = bzx$. Show that all these curves cut orthogonally the family of ellipsoids $x^2 + 2y^2 + 3z^2 = c^2$.

4) Solve $rx = (n - 1)p$

5) Solve $5r + 6s + 3t + 2(rt - s^2) + 3 = 0$

6) Solve the two dimensional Heat Conduction Equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

by method of separation of variables.

7) Check whether the following boundary value problem

$$xy'' + y' + (x^2 + 1 + \lambda)y = 0$$

$$y(0) = 0, \textit{ and } y'(L) = 0$$

L is a constant such that L > 1 is a sturm-Liouville problem or not.

8) Establish Brafman's Generating Function

$$\sum_{n=0}^{\infty} \frac{(c)_n H_n(x) t^n}{(n)!} = (1 - 2xt)^{-c} {}_2F_0\left(\frac{c}{2}, \frac{c}{2} + \frac{1}{2}; -; \frac{4t^2}{(1 - 2xt)^2}\right)$$

9) Prove the recurrence formula

$$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$

Section - C **2 × 16 = 32**

(Long Answer Type Questions)

Note: Answer any two questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10) Solve $r + (a + b)s + abt = xy$ by Monge's Method.

11) A tightly Stretched string with fixed end points $x = 0$ and $x = \pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a initial velocity.

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0.03 \sin x - 0.04 \sin 3x$$

Then find the displacement $\mu(x,t)$ at any point x and at any instance t.

12) State and Prove Euler Lagrange Equation.

13) Find the eigen value and eigen function for the following boundary value problem

$$y'' - 4y' + (4 - 9\lambda)y = 0,\ y(0) = 0,\ y(a) = 0$$

where 'a' is a positive real constant.