MA/MSCMT-02

December - Examination 2019

M.A. / M.Sc. (Previous) Mathematics

Examination

Real Analysis and Topology

Paper - MA/MSCMT-02

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum upto 30 words. Each question carries 2 marks.

1. i. Define σ ring.

 ii. Define Lebesgue measure of a set.

 iii. State Weierstrass approximation theorem.

 iv. Define orthogonal elements.

 v. State Minkowski's inequality.

 vi Define Hilbert space.

 vii. Define normal space.

 viii. Define compact topological space.

Section - B $4 \times 8 = 32$

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2. Let E is a measurable set, then for any set A show that

$$m^*(E \cup A) + m^*(E \cap A) \leq m^*(E) + \mathrm{m}^*(A)$$

3 Show that every bounded measurable function f defined on a measurable set E is L-integrable.

4 If a function is summable on E, then show that it is finite almost everywhere on E.

5 Show that an orthonormal system $\{\phi_i\}$ is complete if it is closed.

6 State and prove Holder's inequality.

7 Prove that in a T_2- space, a convergent sequence has a unique limit.

8 Show that regularity is a topological property.

9 Prove that every closed subset of locally compact space is locally compact.

Section - C $2 \times 16 = 32$

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum upto 500 words. Each question carries 16 marks.

10. Prove that the necessary and sufficient condition for a bounded function f defined on the interval [*a*, *b*], to be L-integrable over [*a*, *b*] for the given $\in > 0$, there exists a measurable partition P of [*a*, *b*] such that

$$U(f, P) - L(f, P) < \in.$$

11. (i) Show that characteristic function of $A \subset X$ is continuous on X if A is both open and closed in X.

    (ii) Show that every metric space is a T_2-space.

12. Prove that outer measure of an interval is its length.

13. State and prove Riesz-Fisher theorem.