MA/MSCMT-01

December - Examination 2019

M.A./M.Sc. (Previous) Mathematics

Examination

Advanced Algebra

Paper - MA/MSCMT-01

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per the given instructions.

Section - A **$8 \times 2 = 16$**

(Very Short Answer Questions)

Note: Answer **all** questions. As per the nature of the question delimit your answer in one word, one sentence or maximum up to 30 words. Each question carries 2 marks.

1) (i) Define conjugate elements in a group.

(ii) Define derived subgroup of a group.

(iii) Define composition series of a group.

(iv) Define sub module.

(v) Define splitting field.

(vi) Define Golois extension.

(vii) Define rank of a matrix.

(viii)Define orthonormal set.

Section - B **$4 \times 8 = 32$**

(Short Answer Questions)

Note: Answer **any four** questions. Each answer should not exceed 200 words. Each question carries 8 marks.

2) If G_1 and G_2 be groups. Let Ni is normal in Gi, $i = 1, 2$. Then show that $N_1 \times N_2$ is normal in $G_1 \times G_2$ and

$$(G_1 \times G_2)/(N_1 \times N_2) \cong (G_1/N_1) \times (G_2/N_2)$$

3) Prove that any two conjugate classes C[a] and C[b] of a group G are either disjoint or identical.

4) If G is a solvable group then show that every subgroup of G is also solvable.

5) Let K/F be a field extension and let $a \in k$ be algebraic over F. Then show that any two minimal monic polynomials for a over F are equal.

6) Let V and V' are vector spaces and $t : V \to V'$ is an isomorphism. Then show that $\{v_1, v_2 ....., v_n\}$ is linearly independent if and only if $\{t(v_1), t(v_2), ..., t(v_n)\}$ is linearly independent.

7) For an $n \times n$ matrix A over a field F prove that $\det(A) = \det(A^T)$

8) Show that the eigen values of a self adjoint linear transformation are real.

9) Let *R* be a Eudidean ring. Then show that every non zero element in *R* can be written as the product of a finite number of prime elements of *R* or is a unit in *R*.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Answer **any two** questions. You have to delimit your each answer maximum up to 500 words. Each question carries 16 marks.

10) Let M be an R module and $N_1, N_2, \ldots N_k$ be sub modules of M. Then show that the following statements are equivalent.

(i) $M = N_1 \oplus N_2 \oplus \ldots\ldots \oplus N_k$

(ii) If $n_1 + n_2 + \ldots + n_k = 0$ then $n_1 = n_2 = \ldots = n_k = 0$ for $n_i \in N_1$

(iii) $Ni \cap (N_1 + \ldots.. + N_{i-1} + N_{i+1} + \ldots... + N_k) = \{0\}$

11) Let F be a field of characteristic zero and let two elements a and b in some extension field of F be algebraic over F. Then show that there exists an element $C \in F(a, b)$ such that $F(c) = F(a, b)$

12) Let *V* and *V′* vector spaces $t : V \to V'$ be linear transformation and V is finite dimensional. Then show that :

$\dim V = \text{rank}\ (t) + \text{nullity}\ (t)$

13) Show that every finite dimensional vector space V with an inner product space has an orthonormal basis.