MA/MSCMT-09

December - Examination 2018

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Section - A **8 × 2 = 16**

(Very Short Answer Questions)

Note: Section A contains 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum words limit is thirty words.

1) (i) Write the conditions for which Laplace transform of f(x) exists.

(ii) Find Laplace transform of y, where $\frac{d^4y}{dx^4} - y = 1$, $y(0) = y'(0) = y''(0) = y'''(0) = 0$.

(iii) Define Fourier Sine transform.

(iv) If $F(p)$ is Mellin transform of $f(x)$ then write Mellin transform of $\int_0^x f(u)\,du$.

(v) Write the relation between Hankel transform and Laplace transform.

(vi) Define Fredholm Integral equation.

(vii) What is resolvent kernel?

(viii) Define orthogonal system of functions.

Section - B **4 × 8 = 32**

(Short Answer Questions)

Note: Section B contains 08 short answer type questions. Examinees have to attempt any four questions. Each question is of 08 marks and maximum words limit is two hundred words.

2) Find Inverse Laplace transform of $L^{-1}\left\{\frac{p}{(p+3)^{7/2}}\ ;t\right\}$

3) Evaluate $\int_0^t \sin u \cos(t-u)\, du$.

4) Solve $(D^2 + 1)y = t \cos 2t$, if $y(0) = 0$, $y'(0) = 0$

5) State and prove Modulation theorem.

6) Solve for $f(x)$, $\int_0^\infty f(x) \cos px\, dx = e^{-p}$.

7) Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation. $g(x) = \sin x + 2\int_0^x \cos(x-t)\, g(t)\, dt$.

8) Convert the following differential equation into Volterra integral equation of second kind $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4 \sin x$; $y(0) = 1$; $y'(0) = -2$.

9) Solve the following integral equation.

$$g(x) = (1 + x^2) + \int_{-1}^{1} (xt + x^2t^2)\, g(t)\, dt.$$

Section - C $2 \times 16 = 32$

(Long Answer Questions)

Note: Section C contains 04 Long answer type questions. Examinees have to attempt any two questions. Each question is of 16 marks and maximum words limit is five hundred words.

10) Prove that $M\{(1+x)^{-a};\ p\} = \dfrac{r(p)\, r(a-p)}{r(a)};\ 0 < Re(p) < Re(a)$.

11) Find the potential $V(r, z)$ of a field due to a flat circular disc of unit radius with its center at the origin and axis along z axis satisfying the differential equation: $\dfrac{\partial^2 v}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial v}{\partial r} + \dfrac{\partial^2 v}{\partial z^2} = 0, 0 \le r < \infty; z \ge 0$

and satisfying the boundary conditions: $V = V_0$ when

$z = 0, 0 \le r < 1,\ and\ \dfrac{\partial v}{\partial z} = 0\ when\ z = 0\ r > 1.$

12) Solve the following integral equation by method of successive approximations. $g(x) = \left(e^x - \frac{1}{2}e + \frac{1}{2}\right) + \frac{1}{2}\int_0^1 g(t)\,dt.$

13) Using Fredholm theory, solve $g(x) = \cos 2x + \int_0^{2\pi} \sin x \cos t\, g(t)\, dt.$