MA/MSCMT-06

December - Examination 2018

M.A./ M.Sc. (Final) Mathematics Examination

Analysis and Advanced Calculus

Paper - MA/MSCMT-06

Time : 3 Hours]   **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C.

Section - A   **8 × 2 = 16**
(Very Short Answer Type Questions)

Note: Section 'A' contains 8 Very short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Define weak convergence of a sequence.

   (ii) Define dual space.

   (iii) State Parseval's identity for a Hilbert space.

   (iv) Define Eigen value and Eigen vector of an operator.

   (v) Write Taylor's formula with Lagrange's remainder.

   (vi) State Lipchitz's function in a Banach space.

(vii) Define Quotient space.

(viii)Define the graph of a function.

Section - B $4 \times 8 = 32$
(Short Answer Type Questions)

Note: Section 'B' contain 08 short Answer Type Questions. Examinees will have to answer any four (4) question. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2)   State and prove Minkowski's inequality for C^n.

3)   If M be a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exist functional F in conjugate space N^* such that $F(M) = \{0\}$ and $F(x_0) \neq 0$.

4)   The mapping $\psi : H \rightarrow H^*$ defined by $\psi(y) = f_y$ where $f_y(x) = (x,y)$, $\forall x \in H$ is an additive, one to one, onto and isometry but not linear.

5)   Show by an example that it is not necessary for an arbitrary operator on a Hilbert space H to possess an Eigen value.

6)   State and prove Global uniqueness theorem.

7)   If T be a linear transformation of a normed linear space N into normed linear space N', then prove that inverse of T i.e. T^{-1} exist and is continuous on its domain iff $\exists$ a constant $K \geq 0$ s.t.

$K\|x\| \leq \|T(x)\|, \forall x \in N$.

8)   State and prove open mapping theorem.

9)   State and prove Schwarz inequality for an inner product space.

Section - C   **2 × 16 = 32**

(Long Answer Type Questions)

Note: Section 'C' contains Four Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to answer in maximum 500 words.

10) Let X be a Banach space over the field K of scalars and let $f : [a, b] \to X$ and g : $[a, b] \to$ R be continuous and differentiable functions such that $\| Df(t) \| \leq Dg\,(t), t \in (a, b)$. then prove that

$$\|f(b) - f(a)\| \leq g(b) - g(a)$$

11) State and prove Inverse function theorem.

12) Let U be an open subset of a Banach space X over K, let $[a, b]$ be a compact interval of R, let f be a continuous function on $U \times [a, b]$ into a Banach space Y over K and let g : $U \to Y$ be defined as

$$g(x) = \int_a^b f(x,t)\,dt, \forall x \in U$$ then prove that g is continuous.

If Df exist as a continuous function on U × [a, b] into $L(X,Y)$ the g is C^1 map and for each $x \in U, Dg(x) = \int_a^b D_1 f(x,t)\,dt$

13) Show that the set of unitary operators on a Hilbert space forms a multiplicative group.

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